Stars Need Benefits: An Experiment on Network Formation

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31 October 2012

PRELIMINARY VERSION – QUOTE AT YOUR OWN RISK

Abstract: It is often found that networked public goods are provided in networks with a core-periphery structure. In such networks, a minority invests in the good and forms the core, while the majority free rides and forms the periphery. Galeotti and Goyal (2010) show that these types of networks may result as a consequence of strategic interaction. In this paper, we investigate the provision of networked public goods in a controlled laboratory experiment. In our 2x2 design, we examine the effects of group size and the presence of (social) benefits for incoming links. We find that group size does not affect the rate of convergence to stable networks, but that social benefits are highly important. The introduction of social benefits leads to more convergence to equilibrium networks and more stable and efficient outcomes. Moreover, the presence of social benefits in large groups leads to the formation of superstars: star networks which are more efficient than Nash stars.

Keywords:

JEL codes:

Acknowledgements: We would like to thank Vincent Buskens and Sanjeev Goyal for their helpful suggestions at early stages of the project and audiences at presentations at the Tinbergen Institute, Amsterdam; the CCC Meeting 2012, UEA, Norwich; the International ESA Conference 2012, NYU, New York and the Workshop on Information in Networks (WIN) 2012, NYU Stern, New York for their comments and suggestions. Financial help by the Research Priority Area Behavioral Economics of the University of Amsterdam is gratefully acknowledged.

1 Introduction

When individuals acquire and share information, they often do so in groups where they are connected in a network structure. Early work by Lazarsfeld et al. (1944) and Katz and Lazarsfeld (1955) suggests that individuals' roles in the network are distributed in a specific way, where a limited number of individuals influences the majority. This has been observed in fields as diverse as fashion, opinions and voting. Similar findings have been reported more recently. On Wikipedia for instance, a small number of 'Wikipedians' is responsible for the vast majority of articles (Voss, 2005; Ortega et al., 2008). These observations imply that information is typically acquired and shared in networks with a core-periphery structure.

In this paper, we investigate what motivates the formation of such core-periphery networks for the acquisition and sharing of information. An example motivating our research is the development of open source software (OSS). On OSS projects, there are usually a few developers that contribute most of the code for a specific program while the majority of those involved are developers and users who contribute little or nothing at all (Lerner & Tirole, 2002; von Krogh & von Hippel, 2006; Crowston et al., 2006). The creation of OSS is a public good: everybody can freely access and change the software code and its use is non-rival.[1] Hence, there are two ways to access software code: one can either write it personally or use someone else's code.

Galeotti and Goyal (2010) show that these patterns may be the result of strategic interaction. In their network formation game, agents desire to access some public good, for instance OSS code, which they can do either by investing personally (writing code) or by making links to others (using someone else's code). The main result in their study is that in every strict equilibrium of the game, the number of players who invest in the public good is limited. These players – 'the influencers' – form the core of the network. Other (periphery) players link to the core, without contributing themselves. Together, the core and periphery players form a core-periphery network.

[1] Depending on the specific open source license, there may be license requirements on open source software. For example, it is often required that the updated software is also made freely available (e.g., Fershtman and Gandal, (2011)).

The GG model provides a useful theoretical structure for our analysis. In a controlled laboratory experiment, we investigate public good provision in a network environment where players decide both on their contributions to the public good and on their network connections. In particular, we will consider two network characteristics that may affect the structure in which individuals choose to acquire and share information. First, we examine whether core-periphery networks, or more specifically star networks, form more easily when players in the core receive (social) benefits from incoming links. Second, we investigate the effect of group size on the networks that are formed. In a 2x2 experimental design, we will systematically vary benefits from incoming links and group size. We vary both in such a way that the equilibrium predictions of the GG model remain unaffected.

Nevertheless, we conjecture that both variations may matter. First, if a player derives (social) benefits from incoming links, her payoff is increasing in the number of players that link to her. In contrast, a player's payoff is independent of the number of incoming links without such social benefits. In the baseline model of GG there are no social benefits. Many applications, though, highlight their role. OSS-developers, for example, may derive positive utility from others using their code. Alternatively, social benefits may represent payments by a third party (Roberts et al., 2006), e.g., through advertisements. The way that we model social benefits allows for either of these interpretations. In the model and in the experiment, the presence of social benefits simply implies that more incoming links will lead to a higher (monetary) payoff.

One feature of the original GG-model without social benefits is that at least one player in the core earns a lower payoff than players in the periphery. This is the case, because in equilibrium all players receive the same benefits from the public good, while the costs of investing are higher than the costs of linking. As a result, any player prefers to be in the periphery of the network. This makes it very difficult to coordinate on a core-periphery network. Moreover, it seems counterintuitive that 'the influencers' earn a lower payoff than the periphery players. Introducing benefits for an incoming link can change this payoff asymmetry without changing the equilibrium network structure. With social benefits, the core players may earn more in equilibrium than the periphery players.

Our second variation is in group size. The GG-model predicts that the maximum number of players who invest and form the core is independent of group size. We conjecture that group size may matter, especially when there are benefits to incoming links. This is because larger networks have more links and therefore generate more benefits from incoming links. For example, the number of peers that use an OSS developer's code may positively influence her status (von Krogh & von Hippel, 2006; Fershtman & Gandal, 2011) or future job opportunities (Lerner & Tirole, 2002). Hence, we may see more players in the core when groups are larger. We empirically investigate this conjecture by varying the group size between four and eight. Our 2x2 design allows us to study the interaction between social benefits and group size.

Our results confirm our conjecture that social benefits matter. When they exist, we observe more Nash network architectures, more convergence to stable outcomes and higher levels of efficiency. In the final 10 rounds, we observe Nash architectures (i.e., a core-periphery structure) in around 75% of the observations in with social benefits. Without social benefits, groups only form Nash architectures in around 20% of the observations in the final 10 rounds. Social benefits also lead to more convergence to stable networks. In the two treatments with social benefits, 11 out of the 14 groups converge to a stable network while without benefits only 3 out of the 13 groups converge to a stable network.

The effect of group size is slightly different than conjectured. It turns out that not the number of players in the core is affected, but their investment in the public good. In groups of four and in the presence of social benefits, groups converge mostly to the Nash star network while with groups of eight we observe that groups mostly converge to '*superstars*'. In a superstar, the core player invests in more units of the public good than in the Nash star. As a result, superstars are more efficient than the Nash star network. We argue that these superstars form because of competition for the core position. With benefits for an incoming link and groups of eight, it becomes very attractive to be in the core of a star. In the first half of the experiment, we observe in most groups that multiple participants compete by investing heavily in the public good after which they converge to a superstar in the second half of the experiment.

We conclude that stars need benefits: the introduction of social benefits has a substantial impact on the rate of convergence. Our main explanation for this effect is that the introduction of social benefits changes the payoff asymmetries between core and periphery players in the network. Social benefits not only lead to more convergence but also to higher efficiency. With social benefits, payoffs are significantly closer to the socially efficient outcome than without social benefits.

The remainder of this paper is structured as follows. We start with a brief discussion of previous studies in section 2. We set up the experimental game in section 3. Section 4 describes the experimental design and procedures and in section 5 we provide equilibrium and efficiency predictions. The results of the experiment are described in section 6 and section 7 concludes.

2 Previous Literature

There is a relatively large theoretical literature on network formation and the provision of public goods in networks, either with endogenously formed networks or exogenously given networks.[2] Most relevant for our study is the work by Galeotti and Goyal (2010), who extend the network public goods game of Bramoullé and Kranton (2007) by adding endogenous network formation using the protocol designed by Bala and Goyal (2000). As mentioned above, we employ the theoretical framework of Galeotti and Goyal (2010) in our experiment.

In this brief overview, we focus on experimental work related to ours. Following the boost in network formation theory, the experimental literature on network formation has been steadily growing in recent years. Closest to our study is a recent working paper by Rong and Houser (2012), who investigate network formation and best-shot public good games. Their work differs both in the game that is played on the network and the treatment variables they use. Their game is a best-shot public good game with binary investment: players either invest or not and in equilibrium the (four) players form a star where only the center invests. Although related, this game is simpler than ours: there is no tension between efficiency and equilibrium as there

[2] For an overview of the theoretical literature on network formation, see for example Goyal (2007) and Jackson (2008). Other theoretical papers that study public good provision on endogenously formed networks include Cho (2010) and Cabrales et al. (2011). Galeotti et al. (2010), Boncinelli and Pin (2012) and Bramoullé et al. (2012) study public good provision on exogenous networks.

exists a unique equilibrium architecture, which is also the efficient outcome of the game. Hence, superstars cannot form in their design and also social benefits and payoff inequity play no role in their setup. In their design, they vary the action space of the players, finding that a restricted action space yields more equilibrium (star) networks.[3]

Within the experimental literature on network formation, the most relevant for our paper is the work that uses the Bala and Goyal (2000) model as the network formation protocol, like we do. Falk and Kosfeld (2003) bring the Bala and Goyal model to the laboratory and find that groups rarely converge to equilibrium (star) networks.[4] In contrast, Goeree et al. (2009) show that star networks do form when players are heterogeneous in terms of their value to others, but not when they are homogeneous. Both Falk and Kosfeld (2003) and Goeree et al. (2009) attribute the (non-)occurrence of Nash equilibria mainly to inequity aversion: in environments where the equilibrium payoff differences between core and periphery players are large, Nash networks are typically not observed.

Rosenkranz and Weitzel (2012) and Charness et al. (2012) analyze public good games with strategic substitutes on a fixed network. Rosenkranz and Weitzel (2012) experimentally investigate the public good game by Bramoullé and Kranton (2007), which is the same game as participants play in our endogenous network setting. They vary the network structure between treatments. The results show that it is very hard for subjects to coordinate in this game, but that on the star and the complete network they find some confirmation of the theory. Charness et al. (2012) test the model by Galeotti et al. (2010), who show that when players have incomplete information about the network structure, the multiplicity of equilibria is reduced compared to the case of complete information. Charness et al. (2012) largely find support for this hypothesis in their lab experiment.

Our results contribute to this literature in the various ways. First of all, we highlight the role of social benefits and payoff asymmetries in network formation and

[3] There are several other experimental papers that investigate games on an endogenous network (Ule, 2005, Corbae & Duffy, 2008, Knigge & Buskens, 2010, Berninghaus et al., 2011, Wang et al., 2012). However, the games they use differ substantially from ours.

[4] Falk and Kosfeld (2003) do observe equilibrium networks in their treatments with ‘one-way flow’, i.e. where information only flows in the direction of the player who maintains the link. We study networks with ’two-way flow’, i.e. information is exchanged between both players on either side of a link. With this protocol Falk & Kosfeld find hardly any convergence to Nash networks.

show that the introduction of social benefits increases the rate of convergence and the overall efficiency. Secondly, we show that superstars may form when group size increases and players jockey to be in the core. Thirdly, we show that the core-periphery networks predicted by the GG-model are observed in the presence of social benefits. Finally, we show that stable star networks may form when the value of a player is endogenous. This extends the work by Goeree et al. (2009) who show that star networks form under exogenous value-heterogeneity but not with homogeneous values.

3 Experimental game

In our experiment, we implement a modified version of GG's 'Law of the Few' model. Our main adaptations are that we introduce *benefits* for an incoming link in some of the treatments and that the investment decision is discrete rather than continuous.[5] In this section, we will set up the version of the game that we use in the experiment. Section 5 describes the equilibrium and efficiency predictions of the experimental game.

Let N denote the set of players $1,2,3,\dots,n$. Every player $i \in N$ decides simultaneously on her links g_i and her investment x_i in some good. Any player i decides on whether to make a link to any of the other players $j \neq i$. If she decides to make a link, we write $g_{i,j} = 1$ and $g_{i,j} = 0$, if not. By convention, we set $g_{i,i} = 0$. The linking decisions of i are summarized by $g_i = (g_{i,1}, g_{i,2}, \dots, g_{i,n})$ and the linking decisions of all players jointly define the (directed) network architecture $g = (g_1, g_2, \dots, g_n)$.

In contrast to the game by GG, investing is a discrete choice. Hence, every player $i \in N$ chooses a positive integer for x_i, i.e. $x_i \in \{0,1,\dots,x_{max}\}$. The strategy of player i is then described by her link and investment decisions, for which we write $s_i = (g_i, x_i)$, and i's strategy space is denoted by S_i. A strategy profile s is the

[5] GG also analyze a model allowing for *transfers*. The main difference with the social benefits in our game is that the size of the transfers in GG equal the costs of making a link, while in our case benefits are strictly smaller than the costs of making a link. Moreover, there is a difference in interpretation; social benefits may be considered to reflect either extrinsic or intrisic motivation, transfers or exogenous income provided by a third party, depending on the application in mind.

collection of all strategies s_i of all $i \in N$. The set of all possible strategy profiles is denoted by S.

Players receive benefits from accessing units of the local public good. The public good is local insofar that any player i accesses her own investment x_i and the investments x_j of her *neighbors* $j \in N_i$. A player j is a neighbor of i, if i links to j or if j links to i, i.e. $N_i = \{j|\max\{g_{i,j}, g_{j,i}\} = 1\}$. The total number of units that i accesses is then given by $y_i = x_i + \sum_{j \in N_i} x_j$. The benefits $f(y_i)$ of accessing units are increasing and concave in y_i. Note that the investment of i and her neighbors are perfect substitutes: i values her own investment the same as any investments by her neighbors.

Investing in units of the good comes at a cost c_i per unit. We introduce small heterogeneities in the cost of investment, as we will describe in more detail in the next section. Making a link comes at a cost k, which is the same for all players and linking is less costly than investing, i.e. $c_i > k \; \forall \; i \in N$. Players receive (social) benefits b from each incoming link. We distinguish between the cases where $b > 0$ and $b = 0$. Moreover, we take $k > b$, which ensures that making links has a net cost to society. All in all, this results in the following payoff function:

(1) $$\Pi_i(s) = f(y_i) - c_i x_i - k \sum_{j \in N_i} g_{i,j} + b \sum_{j \in N_i} g_{j,i}.$$

If we assume self-regarding preferences, a strategy profile s^* is a Nash equilibrium if for every player $i \in N$ it holds that

(2) $$\Pi_i(s_i^*, s_{-i}^*) \geq \Pi_i(s_i, s_{-i}^*) \; \forall \; s_i \in S_i,$$

where $\Pi_i(s_i^*, s_{-i}^*)$ is the payoff of player i given that she chooses s_i^* and the other players choose s_{-i}^*.

4 Experimental design and procedures

As explained above, we employ a full 2x2 design that varies the group size and the presence of benefits for an incoming link. Table 1 summarizes this design: we have groups of either 4 or 8 participants, who play the experimental game either with ($b = 12$) or without ($b = 0$) social benefits.

TABLE 1: 2X2 DESIGN: TREATMENTS AND NUMBER OF GROUPS IN EACH TREATMENT

		Social benefits	
		$b = 0$	$b = 12$
Group size	$n = 4$	7	8
	$n = 8$	6	6

Notes. Rows (columns) distinguish between group size (benefits) treatments. Cell entries denote the numbers of observations (groups) for each combination of treatments.

Previous experimental studies on network formation (Falk & Kosfeld, 2003; Goeree et al., 2009) have shown that network formation games are rather difficult for experimental participants. Subjects need time to understand the game and to coordinate their actions. To deal with these issues, we implemented a partners design: i.e. participants are randomly assigned a role within a group and play the experimental game for 50 rounds, with fixed partners. These partners are identified by letters from A to D or A to H, depending on the group size. The number of rounds is announced in the experimental instructions (see Appendix A).

As introducing public good investment makes the game even more difficult than network games with exogenous values, we introduced two additional measures. First, participants are given 10 practice rounds in order to familiarize them with the game. In each of these rounds, participants can try as many decisions as they like during one minute. There is no interaction in these rounds as the decisions of the other players are exogenously given and kept fixed during the practice round.[6] Second, we introduce small heterogeneities in the costs of investment c_i between players. These differences are small enough to keep the theoretical predictions independent of the player: in equilibrium the cost heterogeneities do not determine who will take which position in a network.

In the network game, all participants simultaneously decide on whom to link to and how much to invest. On their decision screen, participants can review all previous decisions in a history box, where there own decisions are marked in orange and the decisions of others in blue. Once everyone in the session had made a

[6] The practice rounds will be made available online.

decision, participants are informed of the resulting network and their own payoffs. Examples of key screenshots are provided in Appendix B.

In the experiment, earnings are denoted in 'points'. These are exchanged at the end of the experiment at a rate of 1 euro for every 140 points. Subjects are paid for each of the 50 rounds and at the beginning of the experiment they received a starting capital of 1000 points. Table 2 gives the benefits function $f(y)$ (in points) and the costs of linking k and investing c_i are given in Table 3. As specified in section 0, the function $f(y)$ is increasing and concave in y, and $c_i > k \; \forall \; i$. Note that the cost of investing c_i varies by player, where A has the lowest cost of investing and the cost of investing increases by the alphabetical order.

TABLE 2: BENEFITS FROM ACCESSING THE PUBLIC GOOD

Units accessed y_i	0	1	2	3	4	5	5+ℓ
Benefits $f(y_i)$	0	50	80	90	94	96	96+ℓ

Notes: Benefits from accessing units of the public good in experimental points.

TABLE 3: COSTS OF PUBLIC GOOD INVESTMENT AND LINK FORMATION

Role		A	B	C	D	E	F	G	H
Cost per unit of the PG c_i	n = 4	23	25	27	29				
	n = 8	22	23	24	25	26	27	28	29
Link cost k		20	20	20	20	20	20	20	20

Notes: Costs per unit of PG investment and costs per link made in experimental points. At the beginning of the experiment, participants are assigned an id, which is denoted by a letter A to H. This id remains constant throughout the experiment.

Sessions were run in April 2012 in the CREED-laboratory of the University of Amsterdam and lasted about 2 hours.[7] In total, 156 subjects participated in the experiment, each in only one session. Participants were recruited from the local CREED database, which consists mostly of undergraduate students from various fields. Of the participants in our experiments, 39% are female and 60% were studying

[7] We also ran pilot sessions based on a strangers design where the participants were provided with a social history screen. We found that coordination is extremely difficult when participants do not have a fixed role and that with fixed roles, stranger rematching, social history screen and social benefits some groups converge to Nash equilibria, but less often than with partner matching. However, these results are very preliminary and will be addressed in a different paper after more sessions have been run.

at the Faculty of Economics and Business. Cash earnings were between 17.10 euro and 49.50 euro, with a mean of 26.59.[8]

The experiment was computerized using PHP/MySQL and was conducted in Dutch. Upon entering the laboratory, participants were randomly allocated a separate cubicle and communication was prohibited throughout the session. Before starting the network experiment, we elicited risk preferences using a procedure similar to Gneezy and Potters (1997). Participants were only informed of the outcome of this part at the very end of the experiment. After this, participants read the instructions of the network game at their own pace, on-screen. While reading the instructions, a printed summary was handed out to the participants.[9] To ensure that all participants understood the instructions, they were required to answer several test questions. The experiment would only continue if everyone had answered all questions correctly. The ten practice rounds followed the test questions before the network game started.

We ended each session with a short questionnaire where we gathered some demographic data and asked participants to describe how they had made their decisions in the network game. After this, we privately informed participants of the outcome of the risk elicitation task and their aggregate earnings in the experiment. Participants were paid for all rounds of the network game and the risk-elicitation task in cash.

5 Equilibrium and efficiency predictions

Based on the game described in section 3 and the chosen parameters given in section 4 we can derive the one-shot equilibrium predictions for the experimental game. If we assume selfish and rational players, the stage game Nash equilibrium networks of the stage game are the same in all experimental treatments, i.e., they are core-periphery networks where either one or two players invest and form the core while the remaining players link with the core and do not invest themselves.[10] This is also the main result of GG.

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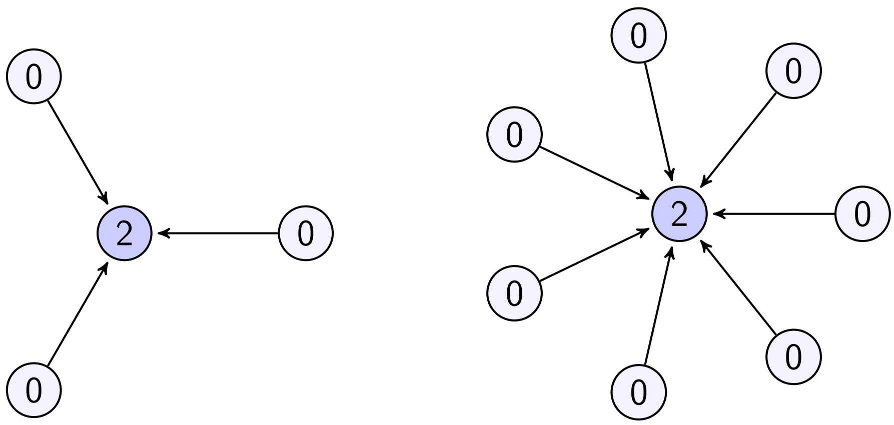
[8] In one session, participants received an additional 10 euro because the session took almost 3 hours. This is not included in the numbers above. They were not informed about this additional lump-sum payment until the experiment had finished.

[9] An English translation of this summary is provided in Appendix C.

[10] We prove that these networks are the Nash equilibria in Appendix D.

Figure 1 and Figure 2 illustrate the Nash equilibrium networks. In these figures, circles represent the players and the numbers inside these circles represent the investment by the player concerned. A link is represented by an arrow, which points away from the player who initiated it. Hence, we see either a *Nash star* network (Figure 1), where only the single core player invests or a *Nash 2-hub* network (Figure 2) where both core players invest. In either case, the other players form links to the core and do not invest.

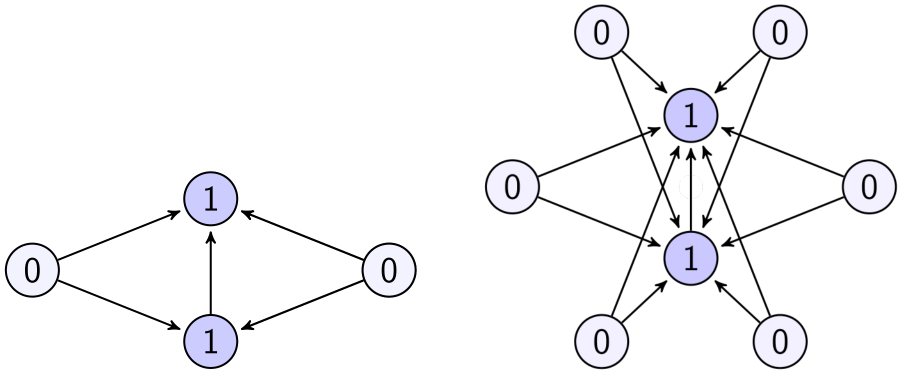
FIGURE 1: NASH STAR NETWORKS



Notes: Nash star networks with n=4 and n=8 players.

Figure 1 shows *Nash star networks* with 4 or 8 players. In either case, there is exactly one player who invests in two units of the public good and makes no links while the other players do not invest and make exactly one link to the core-player who invested in the two units. This type of star network is also referred to in the literature as a *periphery-sponsored star*.

FIGURE 2: NASH 2-HUB NETWORKS



Notes: Nash 2-hub networks with n=4 and n=8 players.

Figure 2 illustrates the Nash core-periphery networks with 2 players in the core, again for groups of 4 and 8 players. We will refer to these networks as *Nash 2-hub networks*. In these networks, each of the two core players invests in 1 unit and one of the two core players makes a link to the other core player, who will makes no links. As in the Nash star, the periphery players do not invest personally but they do link to (both of) the core players.

The intuition behind the proof presented in Appendix D is the following. The marginal benefits of the public good exceed the costs of investing for up to two units of the good. This means, that every player wants to access at least two units of the good. Recall that the investments of a player and her neighbors are perfect substitutes. As the costs of making a link are strictly smaller than the costs of investing, i.e. $k < c_i \; \forall \; i \in N$, any player would strictly prefer to access the investment of others rather than investing personally. Combined, this implies that the total investment will sum up to two units. If there were less investment, all players could strictly improve by investing in additional units. If the total investment exceeds two units, the player(s) who invest can strictly improve by investing less and linking to others. Hence, there will be either one or two players who invest in equilibrium, exactly as is the case in the Nash star and 2-hub networks. Note that there are multiple Nash star and Nash 2-hub equilibria, as every player can be in the core of the network in equilibrium, despite the heterogeneities in investment costs.[11]

[11] To be precise: there are n different Nash star equilibria and $n(n-1)$ Nash 2-hub equilibria.

As mentioned above, the stage game equilibria are the same in all four experimental treatments. GG show that in equilibrium, the number of players in the core is independent of group size. More specifically, they show that the number of core players $|N_c|$ is bounded from above by $|N_c| \leq \hat{y}\frac{c}{k}$, where $\hat{y}$ is the number of units any player will access in equilibrium. Note that this is indeed independent of the group size n. This result carries over directly to our game, which means $\hat{y} = 2$ and $|N_c| \leq 2.9$ as $c_i \in [22,29]\ \forall\ i$ and $k = 20$. Hence, there will be either one or two players in the core of the network.

Introducing social benefits does not affect the set of equilibrium networks. This is the case because, given the strategies s_{-i} of all players $j \neq i$, for any player $i \in N$ a strategy s_i yields a (strictly) higher payoff than strategy s_i' if and only if $\Pi_i(s_i, s_{-i}) - \Pi_i(s_i', s_{-i})$ is (strictly) positive. Hence, player i will prefer strategy s_i over s_i' if

$$(3) \qquad f(y_i) - f(y_i') - c_i(x_i - x_i') - k\left(\sum_{j\in N_i} g_{i,j} - \sum_{j\in N_i'} g'_{i,j}\right) + b\left(\sum_{j\in N_i} g_{j,i} - \sum_{j\in N_i'} g_{j,i}\right) \geq 0.$$

As the linking decisions of all other players are fixed in the above equation, it must be that $\sum_{j\in N_i} g_{j,i} = \sum_{j\in N_i'} g_{j,i}$ and the final term on the left hand side of (3) cancels. Hence, i's decision is independent of the social benefits level b and the set of equilibrium networks must be independent of b.

Next, we consider which networks are efficient in the distinct treatments. We define social welfare w in a network resulting from a strategy profile s simply by the sum of all individual payoffs, i.e. $w(s) = \sum_{i\in N} \Pi_i(s)$. A network s is called *efficient* if

$$(4) \qquad w(s) \geq w(s')\ \forall\ s' \in S.$$

Based on this definition, the efficient network is a (minimally-sponsored) star in which the core player invests in more units than in the Nash star and the periphery players do not invest.[12] If $\mathrm{n} = 4$ the core player invests in 3 units and if $\mathrm{n} = 8$ she invests in 4 units.

Note that in the efficient star networks, the total investment is higher than in the Nash star and that none of the Nash networks is efficient. However, the Nash star is the *efficient equilibrium* as the sum of all benefits and costs of investment is the same in the Nash star and in the Nash 2-hub, but the sum of link costs and social benefits $((k-b)\sum_{i,j\in N} g_{i,j})$ is higher (i.e., more negative) in the Nash 2-hub because there are more links in this network than in a star.

In our analysis, we will also look at *superstars.* A network s is a superstar if it is a periphery-sponsored star where the core invests in more units than in the Nash star. Examples of superstars are illustrated in Figure 3. Note that the efficient periphery-sponsored stars are superstars.

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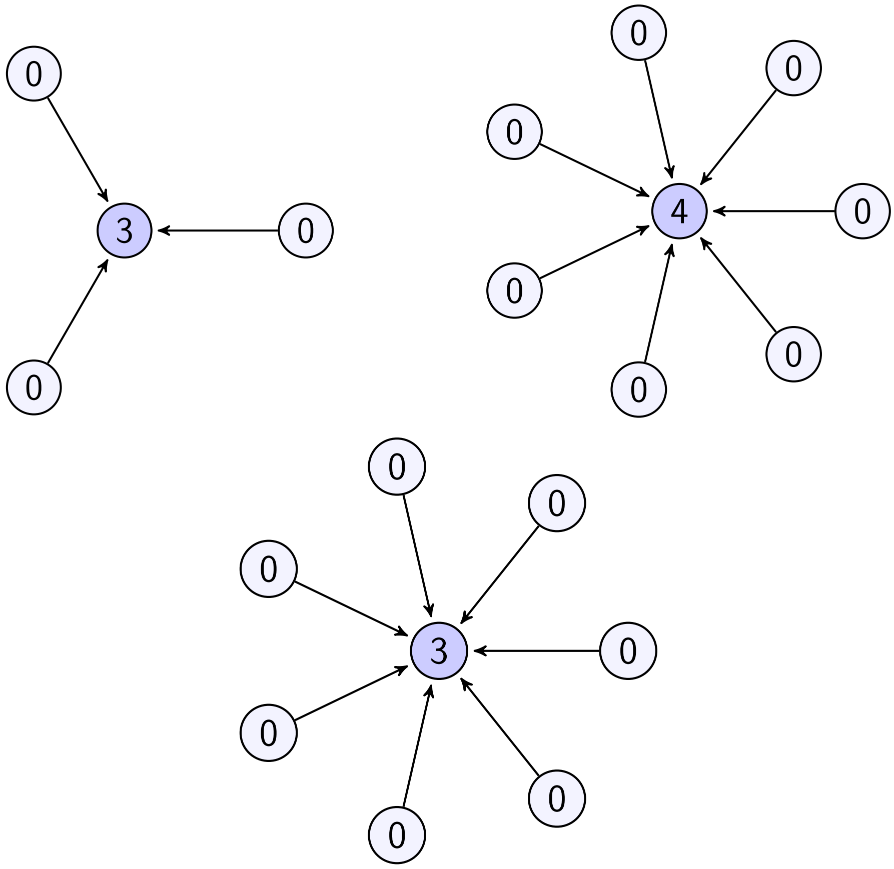
[12] Again, a proof can be found in Appendix D. A minimally-sponsored star is a star network where all connections between the core player and the periphery players are minimally sponsored, i.e. either the core or the periphery player makes a link but not both. Note that the periphery-sponsored star is also a minimally-sponsored star.

FIGURE 3: SUPERSTARS



Notes: Examples of superstars. A superstar is a periphery-sponsored star where the core player invests more than in the Nash star. For $n = 4$, the superstar where the core player invests in 3 units is an efficient outcome of the game, for $n = 8$ this is the superstar where the core-player invests in 4 units.

6 Results

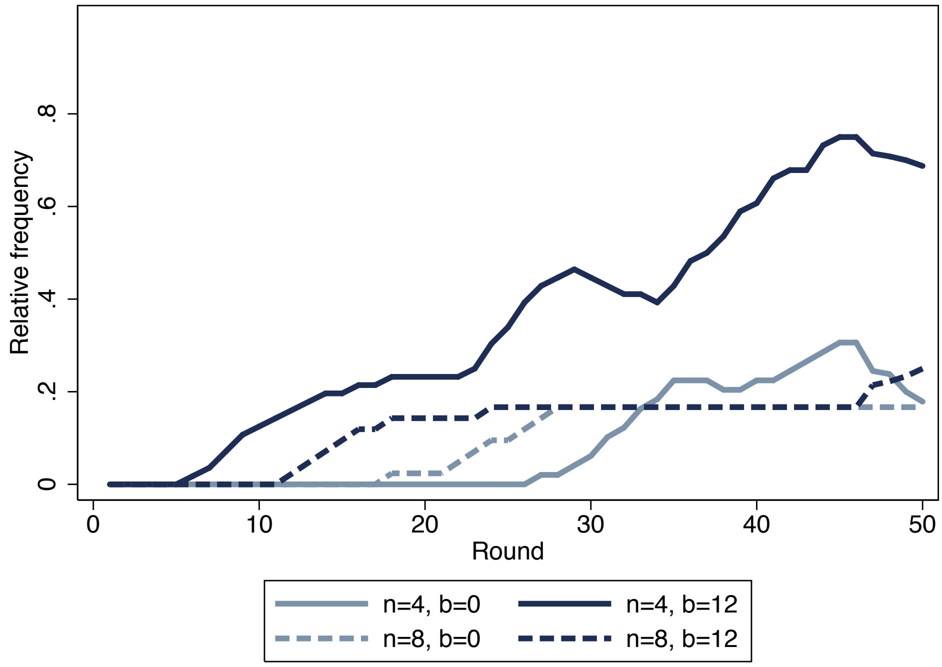
Most of the analysis will present results at the group level. This includes an overview of the networks we observe, and a discussion of the stability and efficiency of these networks. Unless stated otherwise, all tests reported in this section are two-sided Mann-Whitney tests, using a group as the unit of observation. In all tests, the null-hypothesis is that there are no differences between treatments.

6.1 Do groups converge to Nash equilibria?

First, we consider whether Nash networks are formed. Figure 4 shows the relative frequency of Nash networks over time across treatments. The darker lines refer to the treatments with social benefits and the lighter ones to the treatments without them. In addition, the solid lines refer to treatments with groups of 4 while the broken lines refer to treatments with groups of 8.

In the treatment with groups of four and social benefits ($n = 4, b = 12$), we find that the relative frequency of Nash networks steadily increases over time. In early rounds we observe few Nash networks, but in the final 10 rounds of this treatment we observe Nash networks in 70% of the group/round observations.

FIGURE 4: DEVELOPMENT OF NASH NETWORKS



Notes: Lines show the relative frequencies of stage game equilibria by treatment and round: i.e. all Nash star and Nash 2-hub networks. Lines are smoothed by taking the moving average over rounds $t - 3$ to $t + 3$ for every round t.

This is considerably lower in the other three treatments. Again, the number of Nash networks starts out low and increases over time, but now it only increases up to around 20%. If we compare the mean number of Nash networks between the social benefit treatments – ($n = 4, b = 12$) versus ($n = 4, b = 0$) – the differences are significant at the 5% level, both across all rounds ($p = 0.0271$) and for the last 10 rounds ($p = 0.0362$). Also when comparing the treatment with social benefits and larger groups ($n = 8, b = 12$) to the smaller networks with social benefits ($n = 4, b = 12$), we observe significantly fewer Nash networks across all rounds

($p = 0.0372$) and marginally significantly fewer in the last 10 rounds ($p = 0.0843$).[13] Other pairwise differences between treatments are statistically insignificant.

6.2 Do groups converge to Nash architectures?

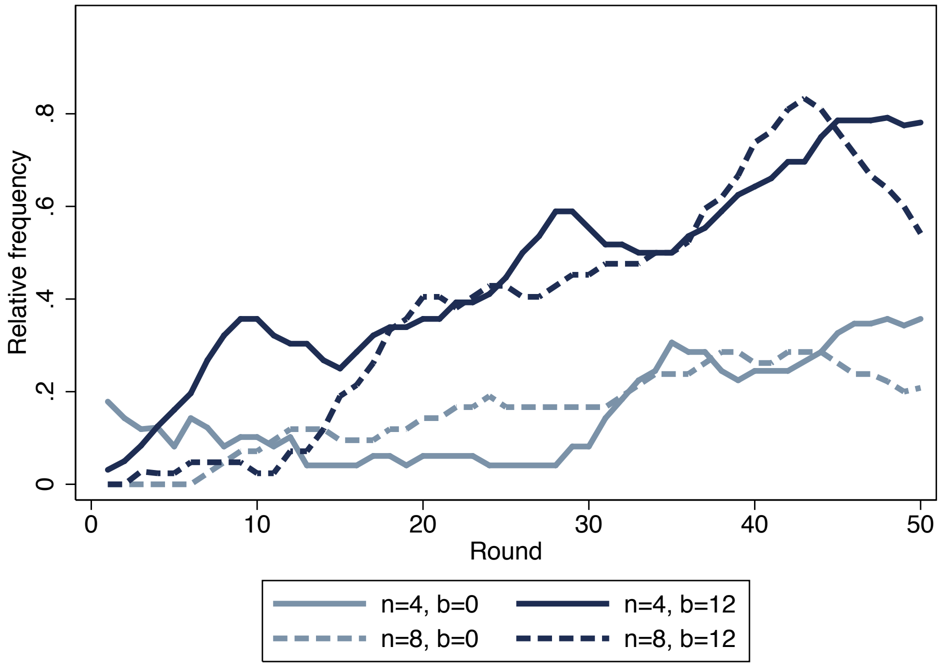
It may be hard for participants to coordinate exactly on Nash networks. Therefore we will first look at *Nash architectures*, which are all networks where the linking decisions are the same as in a Nash network, but the public good investments may differ. This is, we will look at the occurrence of periphery-sponsored star and periphery-sponsored 2-hub networks, irrespective of investment levels.

Figure 5 plots the relative frequency of Nash architectures over time. Again we see that in treatment ($n = 4, b = 12$) the relative frequency steadily increases over rounds, now up to 75% in the last 10 rounds. This is no surprise, as all Nash networks are also Nash architectures by definition. Interestingly, we now also observe that groups converge to Nash architectures in the ($n = 8, b = 12$) treatment.[14] In the last 10 rounds we observe Nash architectures in 72% of the group/round observations in this treatment. For the two treatments without social benefits ($n = 4, b = 0$) and ($n = 8, b = 0$), we observe Nash architectures in only 31% and 25% of the cases, respectively, in the last 10 rounds.

[13] A Kruskal-Wallis test comparing all four treatments also rejects that the relative frequency of Nash networks is the same across treatments: $p = 0.0258$ in all rounds and $p = 0.0659$ in the last 10 rounds.

[14] The upward trend for the two cases with social benefits is confirmed by statistical testing. Two-sided signed Wilcoxon tests reject that the frequency of Nash architectures is the same in the first and second halves of the experiment for the treatments with social benefits ($p = 0.0207$ for ($n = 4, b = 12$) and $p = 0.0340$ for ($n = 8, b = 12$)) but not for the treatments without social benefits ($p = 0.6692$ for (n=4, b=0) and $p = 0.4927$ for (n=8, b=0).

FIGURE 5: DEVELOPMENT OF NASH ARCHITECTURES



Notes: Lines show the relative frequencies of Nash architectures, i.e. all periphery-sponsored stars and periphery-sponsored 2-hub networks. In a Nash architecture, the network structure is the same as in the Nash networks, but the investments may differ from Nash predictions. Lines are smoothed by taking the moving average over rounds $t-3$ to $t+3$ for every round t.

We conclude that groups converge considerably more often to Nash architectures when there are benefits for an incoming link. These differences are also statistically significant. If we compare the treatments with and without social benefits (pooled across group sizes), the difference is significant at the 5% level, both when we look at all rounds ($p = 0.0106$) or at the final 10 rounds ($p = 0.0113$). If we compare the treatments with and without social benefits for a given group size, the differences are significant for groups of four in all rounds ($p = 0.0367$) and marginally so in the last 10 rounds ($p = 0.0568$), For groups of eight, the differences are marginally significant in the final 10 rounds ($p = 0.0934$), but not over all rounds ($p = 0.2207$).

In treatment n=8, b=12, we observe that most groups play a Nash architecture in the final 20 rounds of the experiment. However, unlike treatment $n = 4, b = 12$, we do see an end-effect: there is a sharp decline in the number of Nash architectures in the last 10 rounds. We will look at this end-effect in more detail in section 6.4. Before doing so, we will first investigate to which networks group converge in the different treatments.

6.3 To which networks do groups converge?

In the preceding subsections we found that in the presence of social benefits, groups converge to Nash architectures most of the time. The question remains, to which networks they converge.

Figure 6 shows the proportion of groups that converge to a stable network. A group is said to converge if it is observed to play exactly the same network in more than 5 subsequent rounds.[15] Again, we see that in the two treatments with social benefits, groups converge to some stable network most of the time, while without social benefits there is very little convergence. In the two treatments with social benefits, 11 out of 14 groups converge to a stable network, all of which are periphery-sponsored stars. Without social benefits, only 3 out of 13 groups converge to a stable outcome. Hence, the introduction of social benefits has a crucial effect on the convergence towards stable networks. When we compare these proportions by applying Fisher's exact test, the difference is highly significant ($p = 0.007$, two-sided test).

Aside from the differences in the frequencies of convergence, we observe that groups converge to different networks in distinct treatments. In the treatments with social benefits, groups always converge to periphery-sponsored stars (if they converge). With groups of four this is always the Nash star, while with groups of eight only 1 group converges to the Nash star and 4 groups converge to a superstar. Hence, larger groups converge to star networks that are more efficient than the stage game Nash equilibria. In the treatments without social benefits we observe far less convergence. In ($n = 4, b = 0$) we find that only 2 out of 7 groups converge to a stable network and both groups converge to the Nash 2-hub network. In ($n = 4, b = 0$), only a single group (out of 6) converges, this time to the Nash star.

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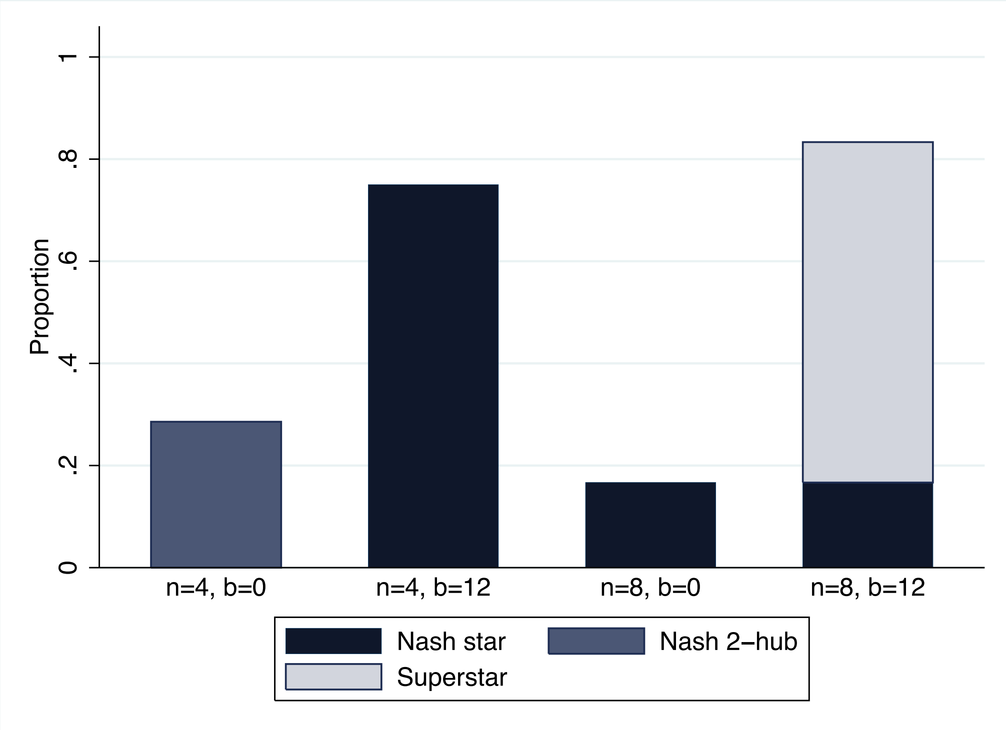
[15] Callander and Plott (2005) and Rosenkranz and Weitzel (2012) use a similar notion of convergence in their analysis.

FIGURE 6: PROPORTION OF GROUPS CONVERGING



Notes: Bars show the proportion of groups converging by treatment and the network to which a group converges. A group converges to a network if this network is observed for more than 5 consecutive rounds. The graph does not change if we define convergence as observing the same network for either more than 3 or 4 rounds. No group converged to any other network and no group converged to more than one type of network.

This shows that stars need benefits: we observe considerably more convergence in the presence of social benefits, both with groups of 4 and groups of 8 players. On the other hand, there is no significant effect of group size on the frequency of convergence ($p = 1.000$, comparing the proportions of groups converging using Fisher's exact test). However, group size does affect the level of public good contributions in the presence of social benefits. With larger groups, the core player invests in more units of the good and superstars form. These superstars are either efficient, or very close to being efficient. We discuss them in more detail in the following subparagraph.

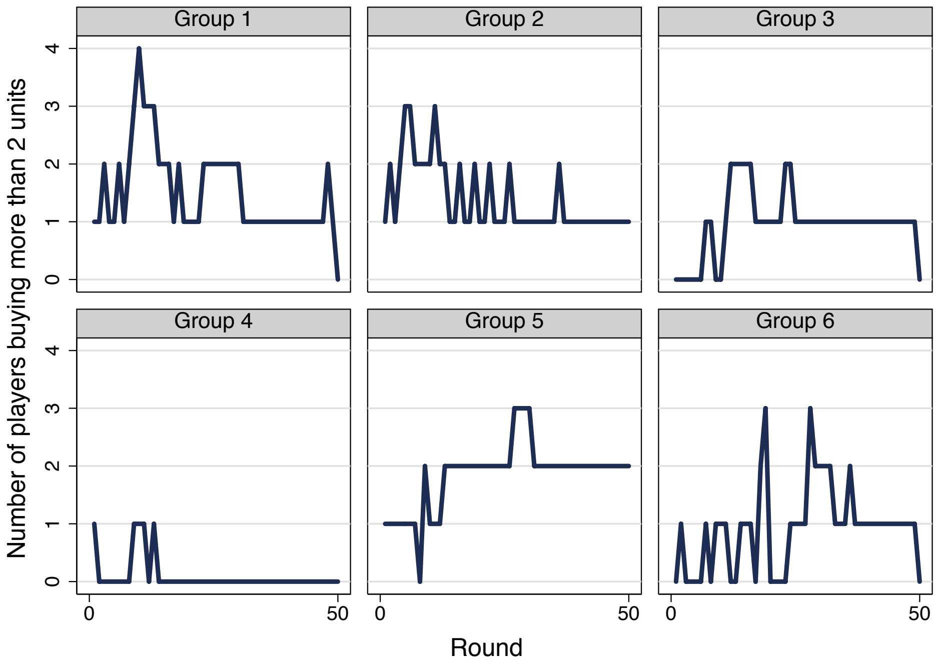
6.4 Superstars

We observed that in treatment ($n = 8, b = 12$), most groups converge to a superstar: a periphery sponsored star where the core invests in more units than in the Nash star.

These star networks are efficient or very close to being efficient. Why do we observe superstars in this treatment? We will discuss two possible explanations.

One possibility is that subjects are concerned about the payoffs of other players; they may for example be inequity averse or altruistic. Recall that in the Nash star with 8 players, the core player earns about twice as much as the periphery players. The core player can decrease this payoff difference by investing in more units of the good: all periphery players will benefit from this while the payoff of the core player drops. Our results do not support such advantageous inequity aversion (guilt) or pure altruism as the reason for the occurrence of superstars, however. In the four groups that converge to a superstar, in the final round three core players decrease their investment in the public good to the Nash level of two, or below. This is illustrated in Figure 7. This shows the number of players who invest in more than two units of the good in each of the six groups in treatment N8B12. Groups 1, 2, 3 and 6 converged to a superstar. In the final round, the core players of the superstars in groups 1, 3 and 6 decreased their investment to two units or less, which they would not do if they experience guilt or altruism.

FIGURE 7: NUMBER OF PARTICIPANTS BUYING ABOVE NASH LEVELS IN $(n = 8, b = 12)$



Notes: Lines show the number of participants who buy more than two units of the good in each of the six groups for every round of treatment ($n = 8, b = 12$).

A second possible explanation is that participants compete for the core position. As noted before, the core position in the Nash star is very attractive in this treatment, which may induce such competition.[16] This is indeed what we observe in our experiment. In all groups we observe some player investing in more than two units of the good, and in five out of six groups we observe multiple participants doing so (cf. Figure 77). In four of these groups, at some point all competitors but one give in and the group converges to a superstar. In one group (group 5), two participants keep up the competition up to the final round.

We only observe superstars in treatment ($n = 8, b = 12$). Moreover, in the other treatments, we rarely observe investment above two units of the good. We note that although superstars are no equilibrium of the stage game, they may be part of a repeated game equilibrium. For instance, all periphery players can credibly threaten the core player by playing a strategy where they move their links to another player if the core player invests in fewer than 3 units.[17] Importantly, the notion of a repeated game equilibrium does not contradict the competition hypothesis, however.

6.5 Frequently observed networks and stability

If a group converges, it is always to a Nash architecture: we observe groups converging to Nash stars, Nash 2-hubs and superstars.[18] These networks are also the most frequently observed networks. Table 4 lists the observed frequency of various

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[16] The benefits function $f(y)$ is such that it is never a best response for a player to link to multiple players if there is one player who invests in two or more units. Hence, the best response for selfish and rational players who do not invest themselves is to make one link to the player with the highest investment.

[17] There are of course many possible repeated game equilibria. It is well beyond the scope of this paper though to provide a complete game-theoretical analysis of the repeated game. Moreover, if we observe convergence in the other treatments, it is always to one of the stage game equilibria. We also do not observe other signs that participants play repeated-game equilibria. For example, we do not observe rotating the center player.

[18] We introduced small heterogeneities in the costs of investment in the public good. Cost heterogeneities alone are not sufficient to achieve coordination: in the treatments without social benefits, we observed convergence only in a small number of groups despite these cost heterogeneities. If convergence is achieved however (in any treatment), it is indeed usually the low-cost role (A) that is in the core of the star. In these treatments, 10 out of 11 groups that converge to a star have the participant in role A in the core, the other group has role D in the core. Over all treatments, in 71% of all the stars role A is in the core, and in 33% of all 2-hub networks A is in the core.

networks. Across all treatments, the most frequently observed network is the Nash star. We observe it especially often in treatment ($n = 4, b = 12$), where it is formed in 140 of the 400 group/round observations. As described in section 6.3, 6 out of 8

TABLE 4: FREQUENCY OF NASH ARCHITECTURES AND THEIR STABILITY

	$n = 4$		$n = 8$	
	$b = 0$	$b = 12$	$b = 0$	$b = 12$
Nash star	0 (-)	140 (0.90)	27 (0.96)	37 (0.97)
Nash 2-hub	31 (0.77)	1 (0.00)	0 (-)	0 (-)
Superstar	0 (-)	4 (0.00)	0 (-)	75 (0.76)
Other star	19 (0.00)	38 (0.13)	19 (0.05)	3 (0.00)
Other 2-hub	5 (0.00)	1 (0.00)	1 (0.00)	0 (-)
Other networks	295 (0.01)	216 (0.04)	253 (0.00)	185 (0.04)
Groups	7	8	6	6
Observations	350	400	300	300

Notes: Cells denote the frequency of the network architectures denoted in the first column in all rounds. The proportion of times that exactly the same network is played in the subsequent round is given between parentheses. The category 'other stars' refers to periphery-sponsored stars only.

groups in this treatment converged to a Nash star and played this network for a substantial number of periods.[19] In treatment ($n = 8, b = 12$) we observe a substantial number of superstars: in 75 out of 300 the group/round observations a superstar is formed. Again, in this treatment most groups converged to a superstar.

Next we look at the stability of different networks. Table 4 also lists the stability of the selected networks by treatment. The Nash star is observed to be very stable in all treatments where it is formed: in more than 90% of the rounds in which it is observed, the Nash star is formed in the subsequent round again. The same is true for the Nash 2-hub network. In treatment ($n = 4, b = 0$), we observe the Nash 2-hub in 31 rounds and in 77% of the subsequent rounds the Nash 2-hub is observed again. In treatment ($n = 8, b = 12$) we frequently observe superstars. They are also very stable: in 76% of the rounds that a superstar is formed it is also observed in the

[19] Obviously, the frequency and the stability of a network are positively correlated.

subsequent round. Note that this is a sizable number: all eight participants are required to make exactly the same decision in the subsequent round. All other

TABLE 5: FIRST ROUND OF CONVERGENCE

		Frequency table	Mean	St. Dev	Groups
$n = 4$	$b = 0$	32 43	37.50	7.78	7
	$b = 12$	9 23 27 38 41 44	30.33	13.26	8
$n = 8$	$b = 0$	25	25.00	-	6
	$b = 12$	21 27 30 39 39	31.20	7.82	6

Notes: Cell entries denote the first round from which the same network was played for more than 5 consecutive rounds, i.e. the minimal round t for which the same network is observed in rounds t to t+5. The column "Frequency Table" lists this minimal t for each of the groups for which convergence was observed.

networks are observed to be far less stable than Nash networks and superstars.

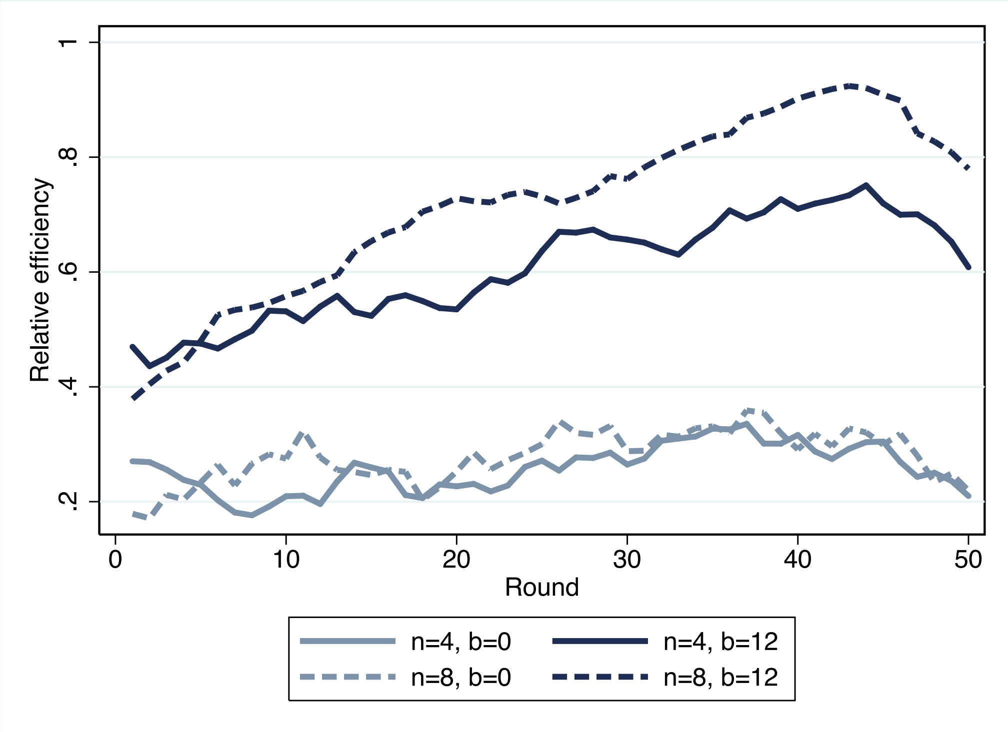
Finally, Table 5 indicates how long it takes for groups to converge. Aside from one group that plays the same network from round 9 to 50, groups usually need a substantial number of rounds to converge to a stable network. Most groups need more than 25 rounds to do so. We conclude that in our experiments groups need time to reach a stable network, but once they reach it they will not easily leave it.

6.6 Efficiency analysis

Next, we consider treatment differences in observed efficiency. We found that the introduction of social benefits yields more convergence towards Nash architectures but does this also imply that we observe more efficient outcomes? Figure 8 shows the mean relative efficiency per treatment over rounds. We define relative efficiency such that it equals 1 if the efficient outcome is reached and 0 if the sum of payoffs equals the sum of payoffs that would be achieved if there were no interaction, i.e. if all players invest in two units and make no links.[20]

[20] Relative efficiency r_t in round t is defined as $r_t(s_t) = (w(s_t) - (\sum_i f(2) - 2c\_i)/(w_{max} - (\sum_i f(2) - 2c\_i))$, where $w(s_t)$ is the sum of all payoffs in a group in round t and $w_{\max}$ is the maximally attainable sum of payoffs. The Nash star, which is the efficient equilibrium of the stage game, has a relative efficiency of between 0.77 and 0.89, depending on the treatment.

FIGURE 8: RELATIVE EFFICIENCY



Notes: Lines show mean relative efficiency across rounds by treatment. Relative efficiency in round t is defined as $(w(s_t) - (\sum_i f(2) - 2c\_i)/(w_{max} - (\sum_i f(2) - 2c\_i))$. Lines are smoothed by taking the moving average over rounds $t-3$ to $t+3$ for every round t.

There is a large difference in relative efficiency between treatments with and without social benefits. Over all rounds, the mean relative efficiency of the treatments with social benefits is 0.67 while this is only 0.27 in the treatments without social benefits. This difference is statistically significant, both when we compare groups of four ($p = 0.0026$) and groups of eight ($p = 0.0039$) with- and without social benefits. Hence, the introduction of social benefits not only leads to more Nash architectures and higher convergence but also to more efficient outcomes.

Figure 8 also suggests that the relative efficiency is higher in ($n = 8, b = 12$) than in ($n = 4, b = 12$). Recall that this is in line with the observation of superstars in the latter treatment and Nash stars in the former. This difference is not statistically significant, however; neither over all rounds ($p = 0.1967$) nor in the final 10 rounds ($p = 0.2432$).[21]

[21] If we run a regression the difference-in-difference estimator (i.e. the coefficient on the interaction effect between group size and social benefits) is also not significantly different from zero ($p = 0.409$).

6.7 Discussion of results

The introduction of social benefits leads to significantly more convergence towards stable outcomes. If we observe convergence with social benefits, it is always to a star network. Why do stars need benefits? A first explanation is that benefits reverse the payoff differences between core and periphery players.[22] In the absence of benefits, the core player earns about half of the payoff of periphery players in the Nash star. As a result, every player would prefer to be in the periphery rather than in the core of a Nash star. Even if players 'see' the Nash equilibrium, they fail to coordinate on who will be the center in the treatments without benefits.

Related to this explanation is the notion that players may be inequity averse. Both Falk and Kosfeld (2003) and Goeree et al. (2009) explain the (non-)occurrence of equilibrium networks to inequity aversion.[23] Goeree et al. (2009) estimate the parameters from the Fehr and Schmidt (1999) model and find that subjects experience envy, but no guilt. This is in line with our results. If we assume that players are envious as in the Fehr and Schmidt (1999) model, and if we use the parameters that Goeree et al. (2009) estimated, the Nash star and the Nash 2-hub are no longer Nash networks in the absence of social benefits.[24] In contrast, the Nash star is robust against envious players when there are social benefits. In the latter case, the periphery players earn less than the core player in the Nash star. However, there is no way for the periphery to decrease the payoff difference, which leaves the Nash star as an equilibrium with envious players. The Nash 2-hub is no longer an equilibrium if we assume that players are envious, even when there are social benefits.

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[22] Without social benefits, the core player in the Nash star earns between 22 and 36 points (depending on the role) and periphery players earn 60 points. With social benefits, the payoffs of the periphery players do not change in the Nash star. The core player earns between 58 and 70 points in groups of four and between 106 and 120 points in groups of eight.

[23] Berninghaus et al. (2006) also highlight the role of inequity aversion in network formation. In their network formation experiment in continuous time, participants often rotate being the core of the star in order to equalize payoffs. We do not observe any behavior consistent with rotating the core position in our data.

[24] We use the parameters that Goeree et al. (2009) estimate, as their game is similar to ours. We assume the following utility function: $U_i(\Pi) = \Pi_i - \frac{\alpha_i}{n-1}\sum_{j\neq i}\max(\Pi_j - \Pi_i, 0) - \frac{\beta_i}{n-1}\sum_{j\neq i}\max(\Pi_i - \Pi_j, 0)$. Goeree et al. (2009) estimate $\alpha_i = 2$ and $\beta_i = 0$: hence participants are only affected by disadvantageous payoff inequity (envy) and not by advantageous payoff inequity (guilt). This also holds for more modest levels of envy. If the core player has $\alpha_i > \frac{1}{5}$, the Nash star is no Nash network and the Nash 2-hub is no Nash network if one core player has $\alpha_i > \frac{1}{2}$ if $n = 4$ and $\alpha_i > \frac{7}{10}$ if $n = 8$.

Another possible explanation of why stars need benefits can be found in the literature on coordination games.[25] Brandts and Cooper (2006) and Brandts et al. (2007) show in weakest-link games that increasing the benefits of coordination leads to coordination at higher effort levels. The analogy can be made to the introduction of benefits. As the sum of payoffs in the Nash networks increase by introducing benefits, one could argue that the costs of non-coordinating increase compared to the situation without benefits. However, in the coordination-game papers the effect was established *within* subjects: participants reacted to a change in the possible loss of miscoordination. Brandts and Cooper (2006) emphasize that it is not the size of the loss but the change itself that matters. This would make it less relevant for our results, because we observe the difference between subjects.

The importance of our results for the cases with social benefits is remarkable in view of the similarity of our results without social benefits to those in other studies. The lack of convergence that we observe in those treatments is in line with results reported in Rong and Houser (2012), who observe Nash networks only in around 13 percent of the cases in their baseline treatments.

The convergence to superstars in cases with benefits and groups of eight also relates nicely to results reported elsewhere. In a natural field experiment, Zhang and Zhu (2011) investigate contributions to Chinese Wikipedia. They use the block of Chinese Wikipedia in mainland China as exogenous variation in group size. They find that contributions are higher in larger groups, which they attribute to social benefits entering the utility function. This is in line with the interaction effect observed in our experiment: in the presence of benefits, larger groups provide more units of the public good. Our experiment and the GG model jointly provide a possible explanation for the findings of Zhang and Zhu (2011). In our experiment, we systematically varied group size and social benefits and showed that the interaction effect between group size and social benefits is driving the increase in contributions. Moreover, we show that this effect may be the result of strategic network formation.

Our results also aid in filling the gap between network formation with homogeneous agents and network formation with heterogeneous agents. Goeree et al. (2009) show that when agents have heterogeneous values, equilibrium networks are

[25] We thank Enrique Fatas for suggesting this explanation.

formed more frequently than when agents are homogeneous. We show that also when agents determine their own value to others (i.e. the public good investments) and are ex-ante homogeneous in this value, participants often coordinate on stable (Nash) networks, but only if there are social benefits related to incoming links.

7 Conclusion

We investigated network formation in an environment where players decide both on their links and their contributions to a local public good. For this, we brought the model of Galeotti and Goyal (2010) to the lab and investigated the effects of group size and social benefits.

We find that social benefits matter. In the presence of social benefits, we observe more Nash network architectures, more convergence to stable outcomes and higher levels of efficiency. We attribute these results to the change in relative payoffs between core and periphery players. As a consequence of this change, the core position is no longer unattractive in groups of four and so attractive in groups of eight that we observe competition for this position. This competition induces public good investment above the levels in the stage game Nash equilibria. These 'superstars' are formed in most groups of eight in the presence of social benefits.

All in all, the theory predicts remarkably well in the presence of social benefits. We observe convergence to stable networks in most groups and convergence is always to a periphery-sponsored star. In groups of four and with social benefits, groups converge to the Nash star. With larger groups, we observe the formation of superstars: periphery-sponsored stars that are more efficient than the Nash star. Without social benefits, however, groups rarely converge to equilibrium networks, and never to any other stable outcome. Moreover, social benefits increase the efficiency of the networks within our laboratory environment. With social benefits, groups form more stable (star) networks, which are efficient or very close to being efficient.

These results are relevant both for theorists and the field. Our experiments indicate that for theory to predict well the payoff differences between players are important. In a model of networked production like the one used here, this implies that social benefits should be included in order to obtain meaningful theoretical predictions. To the best of our knowledge, there is no theoretical work on network formation, which explicitly investigates the role of benefits derived from incoming links. For the setting of our experiment, such theory requires more than a simple reformulation of payoffs, because the equilibria in our setting remain unchanged if social benefits are included. Moreover, most theoretical work neglects the role of social preferences in network formation. This is understandable, insofar that with such preferences, the existing models quickly become terribly complicated. We believe that the development of a behavioral model of network formation that is more tractable would be an important advancement of theory.

For real world environments, our results suggest that policymakers could improve efficiency by introducing or by emphasizing social benefits. For instance, our results predict that OSS developers will contribute more to OSS projects if social benefits are emphasized. Restivo and van de Rijt (2012) provide a nice example of the effect of social benefits in the field. They show that informal rewards ('barnstars') indeed induce contributors on Wikipedia to increase their contributions. Zhang and Zhu (2011) investigate the interaction between group size and social benefits on Chinese Wikipedia and find that larger groups induce higher contributions, just like our results on superstars suggest.

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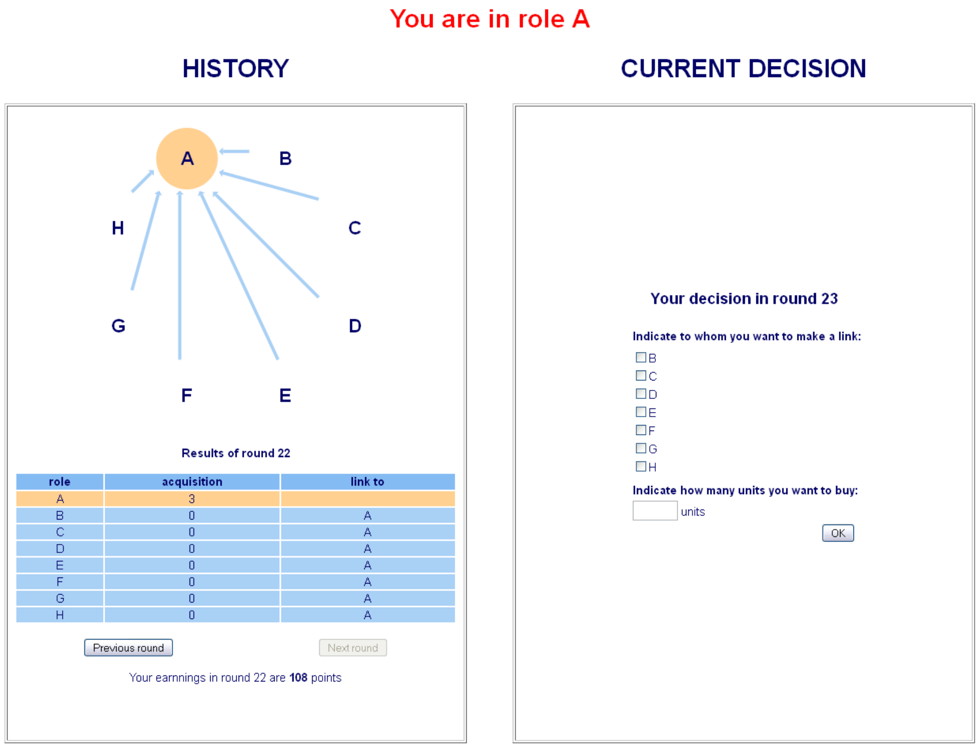
Appendix A: Instructions

Appendix B: Screen shots



Appendix C: Summary of the Instructions

Below is translation of the summary of the (Dutch) instructions, which was handed out on paper to the participants. Any treatment dependent text is marked in red. Full instructions are available upon request.

Summary of the instructions

You can earn points during the experiment. These points are worth money. How many points (and hence how much money) you earn, depends on your own decisions, the decisions of others and luck. At the end of the experiment your earned points will be converted to euros and the earned amount will be paid to you in private. Your total earnings consist of the points you earn in the first part of the experiment (the lottery) and the sum of all points that you earn in the second part of the experiment. At the beginning of the second part you will receive a starting capital of 1000 points. This will also be added to your earnings.

Every 140 points are equivalent to 1 euro.

The second part of the experiment consists of 50 rounds. At the beginning of the experiment, you will randomly placed in a group of 8 participants. The composition of this group will not change during the experiment. In this group you will be randomly assigned a *role*. This role will be indicated by a letter: "A", "B", "C", "D", "E", "F", "G" or "H". The letters "A", "B", "C", "D", "E", "F", "G" and "H" will thus refer to the same participant throughout the entire experiment.

Every round you can earn points by having 'access' to units of a good. The number of points that you earn depends on the number of units that you have access to. This is shown in the following table:

Units	0	1	2	3	4	5	6	7	8	9	10	10+i
Benefits	0	50	80	90	94	96	97	98	99	100	101	101+i

These benefits are the same for each role and all participants in this experiment. The table shows for instance that you earn 80 points if you have access to 2 units and that you earn 100 points if you have access to 9 units of the good.

There are three ways to access units of the good.

1. You buy units of the good yourself.
2. You make a 'link' to another participant. In this case you have access to the units that the other participant has bought and the other participant has access to your purchases. How you make a link will be explained in a moment.
3. Another participant makes a link to you. In this case you have access to the units that the other participant has bought and the other participant has access to your purchases.

If you make a link to another participant and this other participant also makes a link to you, you will only access the entire purchase of the other participant once.

Next to this, you can earn points if other participants make a link to you. For each link that another participant makes to you, you will receive 12 points.

Buying units and making links is costly. The cost of making a link is the same for all roles but the costs of buying units differ by role.

The cost of making a link is 20 points for each role. The cost of making a link are only paid by the participant who makes the link. Every round, you can maximally make one link to each of the other roles. This means that you can maximally make 7 links.

The costs of buying units differ by role. The costs of buying units of the good are given in the table below. Every round, you can maximally buy 5 units of the good.

Role	A	B	C	D	E	F	G	H
Cost per unit	22	23	24	25	26	27	28	29
Cost per link	20	20	20	20	20	20	20	20
Benefits per link to your role	12	12	12	12	12	12	12	12

Appendix D: Proofs

Some additional notation

Most notation was introduced in section 0, but or the proofs we will use the following additional definitions. The marginal benefits of accessing m units of the good are given by $MB(m) = f(m) - f(m-1)$.

A network g is a *core-periphery network* if there are two sets of players $\widehat{N}_c$ and $\widehat{N}_p$ for which it holds that $N_i = \widehat{N}_c \; \forall \, i \in \widehat{N}_p$ and $N_j = N \backslash j \; \forall \, j \in \widehat{N}_c$. In such a network, the players in $\widehat{N}_c$ form the core: any player in the core is linked with all the other players. The players in $\widehat{N}_p$ then form the periphery of the network. Any periphery player is linked with all the core players but with none of the other periphery players. A core-periphery network with a single player in the core is called a *star network* and a core-periphery network with two players in the core is called a *2-hub network.*

We call a link between two players i, j *minimally sponsored* if there exists only one link between i and j, i.e. if $g_{i,j} = 1$ then $g_{j,i} = 0$. A network s or a network architecture g is minimally-sponsored if all the links in the network are minimally-sponsored. We call any core-periphery network *periphery-sponsored* if all the links between the core and the periphery players in a core-periphery network are maintained by the periphery players only.

Nash equilibria of the stage game

The proof closely follows the reasoning of Galeotti and Goyal (2010). Following the theoretical literature on network formation we will restrict our attention to pure-strategy equilibria. We start by stating out version of Lemma 1 in Galeotti and Goyal (2010).

Lemma 1. *In any Nash equilibrium s^*, all players $i \in N$ will access at least two units of the good: $y_i^* \geq 2$ and all players who acquire units personally will access exactly two units of the good, i.e. if $x_i^* > 0$ then $y_i^* = 2$.*

Proof. Suppose that a player $i \in N$ accesses less than two units of the good, i.e. $y_i < 2$. If this is the case i can strictly increase her payoff by investing in units of the good as the marginal benefits $MB(y)$ strictly exceed the marginal costs c_i for $y \leq 2$ and for all $i \in N$. If a player i invests in units personally, i.e. $x_i > 0$, and she accesses more than two units of the good, $y_i > 2$, she can strictly increase her payoff by lowering x_i as $c_i > MB(m)$ if $m > 2$.

Next, we can state our version of Proposition 1.

Proposition 1. *In any Nash equilibrium s^**

1. *the total investment in the network is two, i.e.* $\sum_{i \in N} x_i^* = 2$ *and*

2. *s^* is a core-periphery network where the players in the core do invest in units of the good and the periphery do not invest. Moreover, s^* is a minimally-sponsored network, where all the links between core and periphery players are periphery-sponsored.*

Proof. First we will proof the first statement. Suppose that the condition does not hold and $\sum_{i \in N} x_i^* < 2$. This means that all players $i \in N$ access at most 1 unit of the good, i.e. $y_i^* \leq 1 \; \forall \; i \in N$ and by Lemma 1 we know that every player can increase her payoff by investing in (additional) units of the good. Now suppose $\sum_{i \in N} x_i^* > 2$. By Lemma 1 we know that – in equilibrium – any player who invests in the good will access exactly two units of the good. This would mean, that there exists some player i who invests, i.e. $x_i^* > 0$, and who accesses exactly two units of the good $y_i^* = 2$. As $\sum_{i \in N} x_i^* > 2$, there must be some other player j, who is not in i's neighborhood, i.e. $\exists \; j \notin N_i$, who also invests, i.e. $x_j^* > 0$. As $k < c_i \; \forall \; i \in N$, player i could strictly increase her payoff by linking to j and reducing her investment. Hence, it must be that in equilibrium $\sum_{i \in N} x_i^* = 2$, which proofs the first statement.

From Lemma 1 we know that in equilibrium $\mathrm{y}_\mathrm{i}^* \geq 2 \; \forall \; i$. As $\sum_{\mathrm{i} \in \mathrm{N}} \mathrm{x}_\mathrm{i}^* = 2$, it follows that every player accesses exactly two units of the good: $\mathrm{y}_\mathrm{i}^* = 2 \; \forall \; i$. Then, it must be that any player i who invests, is in the neighborhood of any other player $j \neq i$. This is, if $\widehat{N}_1 = \{i | x_i > 0\}$ is the set of players who invest, then the neighborhood of each player $i \in N_1$ consists of all other players, i.e. $N_i = N \backslash i \; \forall \; i \in \widehat{N}_1$.

In a Nash equilibrium, no player i links to another player j who does not invest, i.e. if $x_j = 0$ then $g_{i,j} = 0 \; \forall \; i \in N$ as making such a link will strictly lower i's payoff. If we define the set of players who do not invest by $\widehat{N}_2 = \{j | x_j = 0\}$, this means that $g_{i,j} = 0 \;\; \forall i \in N, j \in \widehat{N}_2$. This means, that any two players $j, \ell \in \widehat{N}_2$ are not in eachothers neighborhood (i.e. $\ell \notin N_j, j \notin N_\ell \; \forall \; j, \ell \in \widehat{N}_2$). Hence, the neighborhood of any player $j \in \widehat{N}_2$ is given by $N_j = \{i | i \notin \widehat{N}_2\}$ which means $N_j = \widehat{N}_1 \; \forall \; j \in \widehat{N}_2$. Hence, s^* is a core-periphery network where the players who do invest form the core, i.e. $\widehat{N}_c = \widehat{N}_1$, and the players who do not invest form the periphery: $\widehat{N}_p = \widehat{N}_2$. We already showed that the periphery players must maintain all links between core and periphery players only, which implies that these links are minimally-sponsored. It is easy to see that in the two-way flow model we consider all links between core players should also be minimally-sponsored, hence the network s^* must be minimally-sponsored.

Finally, we can define the set of equilibrium networks:

Proposition 3. *The Nash equilibria of the stage game are all:*

1. *Periphery-sponsored stars*
2. *Periphery-sponsored 2-hub networks*

Where the core player(s) acquire in total 2 units of the good, i.e. $x_i^* > 0 \; \forall \; i \in \widehat{N}_c$ *and* $\sum_{i \in \widehat{N}_c} x_i^* = 2$ *and the periphery players acquire no units of the good, i.e.* $x_j^* = 0 \; \forall \; j \in \widehat{N}_p$.

Proof. As $\sum_{i \in N} x_i^* = 2$ and the investment is a discrete choice in integers between 0 and 5, we can have either one or two player who invest, which implies that thw core will consist of either one or two players. In the proof of Proposition 1 we already showed that the equilibrium network is a periphery-sponsored core-periphery network. Hence, any Nash equilibrium is either a periphery-sponsored star or a periphery-sponsored 2-hub network.

Efficient networks

As Galeotti and Goyal (2010) describe in the proof of their Proposition 4, the efficient network is a minimally-sponsored star where only the core player invests. This result carries over directly to the experimental game. Note that given our (small) heterogeneities in c_i, the player with the lowest cost of investing should be the core player. This means that player A is in the core of the efficient star. The efficient level of investment x_A by A is such that the sum of all marginal benefits is still larger or equal than the marginal costs of investing. This is, x_A is set such that it will just satisfy $n \cdot MB(x_A) \geq c_A$. Given the benefits function in Table 2 and the parameters in Table 3, this means that $x_A = 3$ if $n = 4$ and $x_A = 4$ if $n = 8$.