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The Overcharge as a Measure for Antitrust Damages*

Martijn A. Han,[†] Maarten Pieter Schinkel[‡] and Jan Tuinstra[§]

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Abstract

Victims of antitrust violations can recover damages in court. Yet, the quantification of antitrust damages and to whom they accrue is often complex. An illegal price increase somewhere in the chain of production percolates through to the other layers in a sequence of partial pass-ons. The resulting reductions in sales and input demands lead to additional harm to downstream (in)direct purchasers and upstream suppliers to the cartel, respectively. Nevertheless, U.S. civil antitrust litigation is almost exclusively concerned with direct purchaser claims for (treble) damages calculated on the basis of the overcharge. Similar best practice rules are emerging in Europe. In this paper, we show that the direct purchaser overcharge bears no structural relation to the true harm inflicted by a cartel on all of its direct and indirect purchasers and sellers in the chain of production.

JEL-codes: C13, D43, L41.
Keywords: Antitrust, damages, pass-on, overcharge.

1 Introduction

Anticompetitive acts to eliminate competition and prevent new entry can cause severe and widespread harm, for example through inflated prices, reduced quality of service, a smaller product spectrum or retarded innovation. In the U.S., under Section 4 of the Clayton Act, victims of antitrust violations can claim treble damages in civil actions.

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The vast majority of these cases concern cartels. Currently a litigation practice for antitrust damages is developing in Europe.[1]

The identification of antitrust harm can be complicated. The disutilities of poor quality and withheld product improvements are hard to substantiate. Pricing strategies often involve non-linear elements, such as quantity discounts and promotion bonuses. In addition, in longer supply chains, in which one product is an input in the production of the next, antitrust effects can spread. An illegal price increase somewhere in the chain of production percolates through to the other layers in a sequence of partial pass-ons. The resulting reductions in sales cause additional harm to direct and indirect customers and suppliers of the cartel.

In order to determine who is affected by an antitrust violation and to what extent, in principle all actual trades need to be compared to what would have been the market allocation without the anticompetitive behavior—the so-called 'but-for' world.[2] In practice this is often difficult, since it requires information about consumer demand and the structure of the market, such as the number of layers in the production chain, the type and level of competition amongst firms in each layer, their production technologies and costs.

In the U.S., some of these complexities have been constrained by case law. In *American Crystal Sugar* (1952), the courts established the overcharge as the proper basis for quantifying antitrust damage claims of purchasers.[3] According to this method, basic damages—before trebling and interest, if applicable—are calculated as the difference between the anticompetitive cartel price and the competitive but-for price, multiplied by the amount actually purchased. The overcharge ignores lost profits on transactions that could have been made at lower prices.

In *Hanover Shoe* (1968), the Supreme Court ruled against the use of the pass-on defense in Federal antitrust damage actions.[4] In a pass-on defense, the defendant attempts to show that the plaintiff did not in fact suffer the amount of damages claimed on the argument that it was able to pass on all or part of the cartel charges on to it customers. In addition, in *Illinois Brick* (1977) the Supreme Court established that only the direct purchasers have legal standing in Federal court to sue for antitrust damages.[5] *Hanover Shoe* and *Illinois Brick* together cemented the use of the overcharge, which indeed disregards pass through.

Suppliers of a cartel may also be harmed and in *Mandeville Island Farms v. Sugar* (1948), the Supreme Court held that growers of sugar beets could maintain a treble-damages action against refiners who had alledgedly conspired to fix the price that they would pay for the beets.[6] However, in the 1970s a number of supplier suits for antitrust damages failed because circuit courts denied standing to classes of employees and suppliers of cartels.[7] In 1983, the Supreme Court closed the door on upstream

[1] See, *e.g.*, Commission of the European Communities (2008).

[2] See Fisher (2006) and van Dijk and Verboven (2006) for a survey of some of the methods that can be applied in the determination of but-for prices.

[3] *American Crystal Sugar Co. v. Mandeville Island Farms,* 195 F2d 622 (9th Cir. 1952).

[4] *Hanover Shoe Inc. v. United Shoe Machinery Corp.* 392 U.S. 481 (1968).

[5] *Illinois Brick Co. v. Illinois* 431 U.S. 720 (1977). See also Schinkel *et al.* (2008).

[6] *Mandeville Island Farms, Inc. v. American Crystal Sugar Co.*, 334 U.S. 219 (1948).

[7] In *Contreras v. Grower Shipper Veg. Ass'n of Cent. Cal.*, 484 F.2d 1346 (9th Cir. 1973) the

antitrust damages cases in *Associated Contractors v. Carpenters*, in which a class of carpenters sought antitrust damages for business loss resulting from the contractors association using anticompetitive means to work around their union.[8] While the Court agreed that the association's acts might have violated the antitrust laws, it concluded that since the carpenters were neither a consumer nor a competitor of the contractors, the union's allegations of consequential harm were indirect and therefore insufficient as a matter of law.

As a result of these various legal constraints, in the vast majority of U.S. antitrust damages actions, the plaintiffs are direct purchasers and their claim is based on the overcharge.

In this paper, we consider the effects of a cartel somewhere in a chain of production with an arbitrary number of layers. Competition in each layer is specified between perfect competition and monopoly. This allows us to exactly characterize the effects of the cartel's direct and indirect purchasers, as well as its direct and indirect suppliers. We assess the bias introduced by relying on the overcharge on the direct purchasers—which we refer to as the 'direct purchaser overcharge'—for the estimation of the actual antitrust harm in the chain.

We find that even in the most basic of settings—with unit pricing and input price taking—the direct purchaser overcharge is a poor measure of the true antitrust harm. The overcharge can grossly underestimate the actual antitrust harm, depending on such characteristics of the market as the shape of demand, the number of producers, the type of competition, and the location of the cartel in the chain of production. In particular, we show that lost profit harm ignored by the direct purchaser overcharge may increase without bound with the length of the production chain. Moreover, the method misses harm sustained upstream from the cartel, which can be substantial. The ratio of antitrust harm to the direct purchaser overcharge can be anything between one and infinity.

The existing literature on cartel pass-on effects uses a model with only three layers: a top layer of input producers that form a cartel upstream, a layer of direct purchasers downstream who sell to a third layer of final consumers.[9] Hellwig (2006) shows that the deadweight-loss of a direct purchaser monopolist from discrete cartel price increases is equivalent to the part of the overcharge it was able to pass on to consumers. On this basis, Hellwig argues that the direct purchaser overcharge is a good measure for the actual antitrust harm sustained by this group. Verboven and van Dijk (2007) use the mainstream model to analyze an infinitessimal cartel price increase to determine 'discounts' to be given on the direct purchaser overcharge to correct for pass-on to consumers and output effects locally. Basso and Ross (2007) extend the approach to differentiated products, so that there can be input substitution, to produce a numerical table of correction factors for a discrete cartel price increase.

9th Circuit Court denied a class of employees laid off by an alleged price-fixer standing to sue for antitrust damages. In *Comet Mechanical Contractors, Inc v. E.A. Cowen Constr. Inc*, 609 F.2d 404 (10th Cir. 1980) a subcontractor could not seek compensation for reduced demand for its services resulting from alleged bidrigging by general contractors. See Page (1985), p.1492.

[8] *Associated Gen. Contractors of Cal., Inc. v. Carpenters,* 459 U.S. 519 (1983).

[9] See Harris and Sullivan (1979) and Kosicki and Cahill (2006).

For all practical purposes, Boone and Mueller (2008) express the share of otherwise (unspecified) total antitrust harm borne by consumers for an infinitessimal price increase as a function of such magnitudes as the HHI, the PCM, quantities, costs and elasticities.

This paper is organized as follows. In Section 2, we decompose the various welfare effects caused by a cartel anywhere in a chain of production and relate aggregate and individual effects to the overcharge on the direct purchasers. In Section 3, we evaluate the direct purchaser overcharge as an estimator for antitrust harm. Section 4 concludes. Derivations of intermediate results and proofs are collected in an appendix.

2 Cartel Effects in a Chain of Production

2.1 A Vertical Model of Production

Consider a vertical chain with several layers of intermediaries, each adding value to produce a homogenous consumer product. Let consumer demand for the final product be represented by an inverse demand function $P : \mathbb{R}_+ \to \mathbb{R}_+$, which is nonincreasing, twice differentiable and continuous in aggregate production Q. Let there be K layers of production, with n_k firms active in layer k. Except for layer 1, where the raw materials originate, the firms in any layer k each transform a homogeneous input they purchase from firms in layer $k-1$, using a one-to-one technology, into a homogeneous new output, which they sell on to the firms in layer $k+1$. Eventually, the firms in layer K sell the final product to consumers. Figure 1 illustrates.

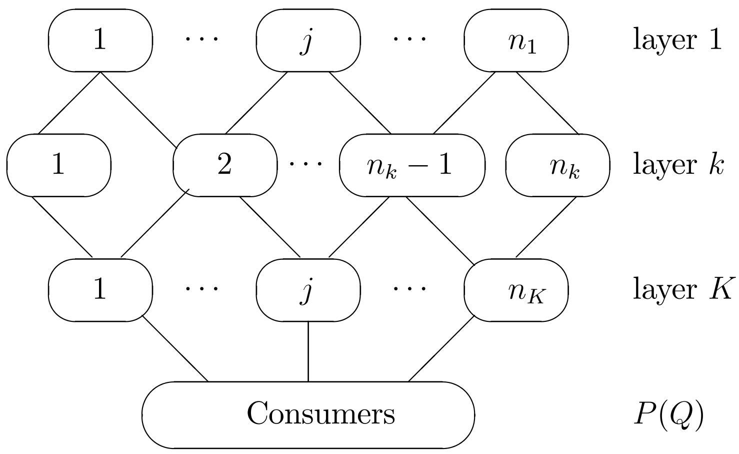

Figure 1: A longer vertical chain of production.

We assume that the number of firms in each layer is exogenously given and fixed. The cost function for firm j in layer k is given by $p_{k-1}q + c_{jk}(q)$, where p_{k-1} is the unit price for the input from layer $k-1$ (with $p_0 = 0$), and $c_{jk}(q)$ are the costs for transforming q units of the input into q units of the output. We abstract from nonlinear pricing or more general types of vertical relations between firms from different layers.

Firms move simultaneously within the same horizontal layer, and sequentially following the layer above—so layer 1 moves first, layer 2 second, and so on. That is, in keeping with the literature on cartel pass-on referred to in the introduction we assume that all firms in each layer purchase from the producers in the layer above at going prices, without bargaining. The market equilibrium is found by backward induction. First consider layer K. For any possible input price p_{K-1}—and given final consumer demand $P(Q)$—we can determine the resulting equilibrium output in layer K. Let the relationship between this equilibrium quantity and p_{K-1} be represented by a uniquely defined, nonincreasing, continuous and differentiable function $p_{K-1}(Q)$. This function serves as the inverse demand function for the firms in layer $K-1$.[10] Firms in layer $K-1$ then determine, for any p_{K-2}—and given their inverse demand function $p_{K-1}(Q)$—their optimal production quantity, in turn leading to an inverse demand function $p_{K-2}(Q)$ for firms in layer $K-2$, and so on.

For analytical convenience, we analyze a model with conjectural variations to simulate various types of competition in each layer.[11] That is, in each layer k firm j's conjecture about the reaction of the other firms in that layer to its quantity decision is $\vartheta_k = \frac{\partial Q}{\partial q_{jk}}$. We assume that ϑ_k is the same for all firms in a horizontal layer, but may be different for firms from different layers vertically. Hence, given input price p_{k-1}, the first-order condition for a symmetric equilibrium in layer k, with conjectural variations parameter ϑ_k is

$$p_k(Q) + \frac{\vartheta_k}{n_k} Q p_k'(Q) - c_k - p_{k-1} = 0. \tag{1}$$

Note that the classic Cournot conjecture corresponds to $\vartheta_k = 1$. If $\vartheta_k = 0$, all firms in layer k are price takers, so that in equilibrium prices will equal marginal costs, *i.e.*, $p_k = p_{k-1} + c_k$. The specification $\vartheta_k = n_k$ is analytically equivalent to full horizontal collusion in layer k. Other values of ϑ_k close to n_k can be interpreted as forms of imperfect collusion, in which joint-profit maximization is further constrained, for example when the cartel members understand that the risk of discovery and the size of a consequential damage claim are likely to depend upon the cartel's pricing and production strategy.[12]

We denote the ultimate equilibrium quantity on the inverse demand function $p_1(Q)$ faced by the firms in layer 1 by Q^*. Equilibrium prices then clear as $p_1^* = p_1(Q^*)$, $\ldots$, $p_{K-1}^* = p_{K-1}(Q^*)$ and $p_K^* = P^* = P(Q^*)$. The individual level of production of firm j in layer k equals q_{jk}^*, with $\sum_{j=1}^{n_k} q_{jk}^* = Q^*$. Equilibrium profits and consumer surplus follow straightforwardly from these quantities and prices.

Now suppose that the firms in some layer $g \in \{1, \ldots, K\}$ form a cartel, while competitive conditions in all other layers remain the same—note that these may

[10] This is an assumption in so far that *a priori* it can be that for a certain value of p_k there exist multiple equilibria in the quantity-setting subgame in layer k. For all specifications considered in this paper, however, an explicit, continuous and differentiable relationship between p_k and Q exists for every k.

[11] Basso and Ross (2007) takes the same approach. For a conceptual critique of conjectural variations, see Hahn (1989).

[12] Salant (1987) and Harrington (2004) offer explicit analyses of some of the effects of such additional incentive constraints.

include pre-existing cartels elsewhere in the chain. Our setup implies that the cartel uses its obtained market power to raise unit prices *vis-a-vis* its customers, but remains a price-taker on the market for its inputs. We further assume that there are no cartel-specific efficiency gains that would somehow allow the cartel to produce at lower costs than its members could in competition. Let the resulting equilibrium quantity and equilibrium prices under the cartel regime be denoted by Q^g and p_k^g for $k = 1, \ldots, K$.[13] Firm j in layer k produces q_{jk}^g with $\sum_{j=1}^{n_k} q_{jk}^g = Q^g$, for all k.

2.2 Decomposition of Cartel Effects

The presence of the cartel causes harm to welfare in the form of high unit prices, resulting in lost profits throughout the chain of production, lost consumers surplus, and deadweight-losses, while the cartel members raise their profits. That is, $Q^g < Q^*$ and $p_g^g > p_g^*$. Typically also $p_k^g > p_k^*$ for $k > g$ and downstream intermediaries and consumers are harmed by the price conspiracy. Under certain specifications, the profits of some downstream intermediaries—in particular direct purchasers—may actually increase in response to the upstream price increases.[14] Also, prices higher up in the chain may either in- or decrease, depending upon the shape of demand and cost functions. *In toto*, however, collusion on unit prices is bad for welfare.

The impact of the g-level cartel's unit price increase on one particular layer k of production that is downstream from layer g can be decomposed into three distinct effects. The *overcharge effect* on layer k is the amount by which the firms in this layer are overcharged by the previous layer $k-1$. Part of the burden of this overcharge may be passed on by the firms in layer k to the next layer of production, layer $k+1$. This is the *pass-on effect*. Finally, the *output effect* results from the decrease in production due to the cartel. It amounts to the losses in profits from the reduction in sales.[15] We consider each of these effects separately, as they are borne out in lost profits.

Consider the aggregate profits of firms in layer k. In the competitive benchmark, these are $\pi_k^* = \left(p_k^* - p_{k-1}^*\right) Q^* - \sum_{j=1}^{k} c_{jk}\left(q_{jk}^*\right)$. Under the cartel regime, they are $\pi_k^g = \left(p_k^g - p_{k-1}^g\right) Q^g - \sum_{j=1}^{k} c_{jk}\left(q_{jk}^g\right)$. The difference $\triangle\pi_k = \pi_k^* - \pi_k^g$ can be decomposed

[13] In our model it is optimal for the cartel to increase prices symmetrically, so that the cartel price p_g^g is the same for each direct purchaser. Verboven and van Dijk (2007) also consider various asymmetric cartel price mark-ups in their reduced form model. This could be relevant for example if some of the direct purchasers are integrated with a colluding firm, however Verboven and van Dijk (2007) do not derive mark-up differentiation as an optimal pricing strategy.

[14] For an analysis of comparative statics effects in Cournot models, see Dixit (1986) and Quirmbach (1988).

[15] Our decomposition follows Hellwig (2006), but we use a slightly different terminology. Where he distinguishes between a *direct cost effect* and a *business loss effect*, we use *overcharge effect* and *output effect*, respectively. Verboven and van Dijk (2007) also analyse both effects, but for exogenous infinitessimal changes in the input prices, rather than endogenous discrete equilibrium effects. They speak of *direct cost effect* and *output effect*, respectively. All three papers share the definition of the *pass-on effect*.

as follows

$$\begin{aligned}\triangle\pi_k &= Q^g\left(p_{k-1}^g - p_{k-1}^*\right) - Q^g\left(p_k^g - p_k^*\right) \\ &\quad + \left[\left(Q^* - Q^g\right)\left(p_k^* - p_{k-1}^*\right) + \sum_{j=1}^{n_k} c_{jk}\left(q_{jk}^g\right) - \sum_{j=1}^{n_k} c_{jk}\left(q_{jk}^*\right)\right] \\ &= \xi_k - \omega_k + \sigma_k.\end{aligned} \tag{2}$$

The first factor is the overcharge-effect on firms in layer k, thus defined as

$$\xi_k = Q^g\left(p_{k-1}^g - p_{k-1}^*\right), \tag{3}$$

or the price increase of the product of the previous layer $k-1$, multiplied by the quantity purchased under the cartel regime.

The second factor,

$$\omega_k = Q^g\left(p_k^g - p_k^*\right), \tag{4}$$

corresponds to the pass-on effect, which is the amount of the price increase that layer k passes on to layer $k+1$. It is equal to the price increase of layer k multiplied by the quantity produced under the cartel regime. Note that ω_k, the pass-on effect of layer k, equals the overcharge effect suffered by layer $k+1$, that is $\xi_{k+1} = \omega_k$.

The last factor in equation (2) is the output effect,

$$\sigma_k = \left(Q^* - Q^g\right)\left(p_k^* - p_{k-1}^*\right) + \sum_{j=1}^{n_k}\left(c_{jk}\left(q_{jk}^g\right) - c_{jk}\left(q_{jk}^*\right)\right). \tag{5}$$

This part represents the loss of profits that could have been made on the larger volume in the competitive benchmark. It can be rewritten as the sum of individual firm output effect, $\sigma_k = \sum_{j=1}^{k} \sigma_{jk}$, with

$$\sigma_{jk} = \left(q_{jk}^* - q_{jk}^g\right)\left(p_k^* - p_{k-1}^* - \bar{c}_{jk}\left(q_{jk}^*\right)\right) + q_{jk}^g\left(\bar{c}_{jk}\left(q_{jk}^g\right) - \bar{c}_{jk}\left(q_{jk}^*\right)\right).$$

Here $\bar{c}_{jk}\left(q_{jk}\right) = c_{jk}\left(q_{jk}\right)/q_{jk}$ are the average costs for firm j in layer k, evaluated at q_{jk}. The first part of the individual output effect, $\left(q_{jk}^* - q_{jk}^g\right)\left(p_k^* - p_{k-1}^* - \bar{c}_{jk}\left(q_{jk}^*\right)\right)$, equals the lost sales times the average profit margin and is always positive. The sign of the second part, $q_{jk}^g\left(\bar{c}_{jk}\left(q_{jk}^g\right) - \bar{c}_{jk}\left(q_{jk}^*\right)\right)$, is ambiguous. It is positive (negative) if average costs for firm j in layer k are decreasing (increasing).

The effects on layers *upstream* from cartel layer g—*i.e.*, layers $k < g$—can be decomposed in much the same way. These layers also each face an overcharge, a pass-on and an output effect. The effect of the cartel on upstream prices results from reduced derived demand and are ambiguous. As a result, so are the signs of the upstream overcharge and pass-on effects. If all upstream prices increase, the upstream overcharge and pass-on effect are positive. If all upstream prices decrease, both the overcharge effect and the pass-on effect of the upstream layers will be negative, corresponding to a decrease in input costs and a decrease in revenues, respectively. In certain specifications, it may also be that all upstream prices remain the same so that there are no upstream overcharge and pass-on effects. Generally, some of the upstream prices may increase and others decrease.

Finally, consider the loss in consumer surplus of the final consumer. It is given by

$$\triangle CS = CS^{*} - CS^{g} = \xi_C + \sigma_C,$$

with

$$\xi_C = Q^{g}\left(p_K^{g} - p_K^{*}\right) \text{ and } \sigma_C = \int_{Q^{g}}^{Q^{*}} \left[P\left(Q\right) - P\left(Q^{*}\right)\right] dQ,$$

where $p_K^{g} = P\left(Q^{g}\right)$ and $p_K^{*} = P\left(Q^{*}\right)$. Note that, since these are the final consumers, there is no pass-on effect. Also note that, because $Q^{g} < Q^{*}$, both ξ_C and σ_C are strictly positive and final consumers unambiguously suffer from the cartel.

Figure 2 illustrates the various typical effects identified in a model with three production layers ($K = 3$) and final consumers.

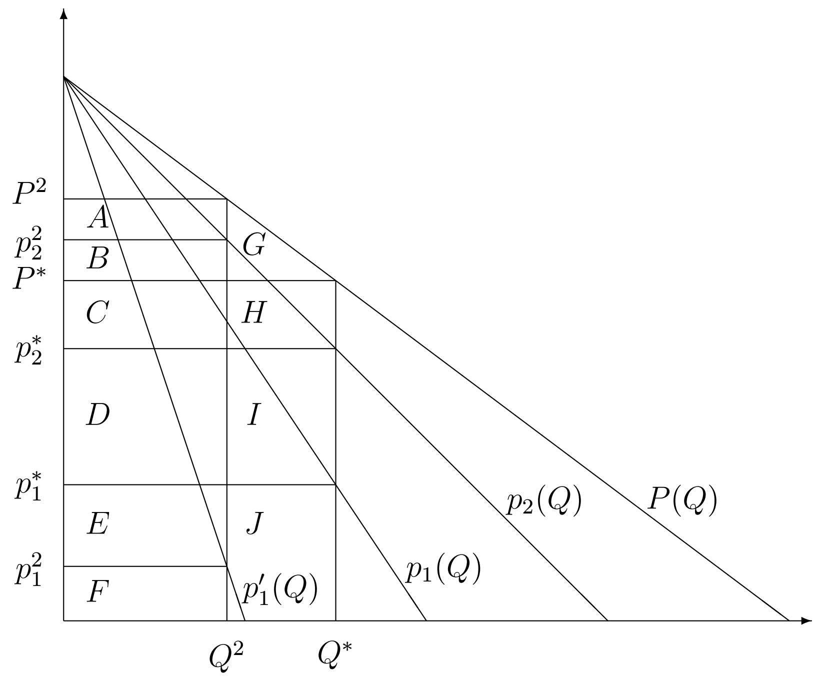

Figure 2: Decomposition of antitrust harm in a three layer model.

We assume that marginal own production costs are constant and equal to zero for all firms, *i.e.*, $c_{j1}\left(q\right) = c_{j2}\left(q\right) = c_{j3}\left(q\right) = 0$, for all j and every q. Given inverse consumer demand $P\left(Q\right)$, the implied inverse demand for the product of firms in layer 2 is given by $p_2\left(Q\right)$. Competition in layer 2 results in inverse demand function $p_1\left(Q\right)$ for the firms in layer 1. Their competitive benchmark is given by the quantity Q^{*} and prices P^{*}, p_2^{*} and p_1^{*}, respectively. Now suppose firms in the second layer collude. This leads to a reduction in the quantity they supply. That is, for every price p_1, layer 2 demands less of the input supplied by layer 1, resulting in an inwards shift of

$p_1(Q)$ to $p_1'(Q)$. Under the cartel regime, output decreases to Q^2, and equilibrium prices become P^2, p_2^2 and p_1^2, respectively. Note that in this illustration we assume that p_1 decreases under the cartel, which need not be the case.

It is insightful to identify some of the areas in the graph. The loss in profits of the direct purchasers of the cartel (layer 3) equal

$$\triangle\pi_3 = \pi_3^* - \pi_3^2 = C + H - A = (B + C) - (A + B) + H = \xi_3 - \omega_3 + \sigma_3,$$

with $\xi_3 = (p_2^2 - p_2^*)\,Q^2 = B + C$ being the amount by which the firms in layer 3 are overcharged, $\omega_3 = (P^2 - P^*)\,Q^2 = A + B$ the amount passed-on to the final consumers, and $\sigma_3 = H$ the output effect. The loss in consumer surplus is

$$\triangle CS = CS^* - CS^2 = (A + B) + G = \xi_C + \sigma_C,$$

where $\xi_C = A + B = \omega_3$ is the overcharge imposed by layer 3, and $\sigma_C = G$ is the output effect for consumers. Profits of the (direct) suppliers to the cartel change by

$$\triangle\pi_1 = \pi_1^* - \pi_1^2 = E + F + J - F = E + J = -\omega_1 + \sigma_1,$$

where $\omega_1 = -E$ is the pass-on from layer 1 to layer 2, which is negative since p_1 has increased. Note that there is no overcharge for layer 1. Its output effect is given by $\sigma_1 = J$. Finally, consider the colluding layer 2. Profits of the cartel members change by

$$\triangle\pi_2 = \pi_2^* - \pi_2^2 = D + I - (B + C + D + E) = -E - (B + C) + I = \xi_2 - \omega_2 + \sigma_2,$$

where $\xi_2 = -E = \omega_1$ is the overcharge from their direct suppliers, $\omega_2 = B + C$ is the pass-on to the next layer and σ_2 is the output effect. Obviously, the sum total of these effects is negative or otherwise it would not be profitable for the cartel to form. Notice also that

$$\triangle\pi_1 + \triangle\pi_1 + \triangle\pi_3 + \triangle CS = G + H + J + I = \sigma_1 + \sigma_2 + \sigma_3 + \sigma_C,$$

that is, the sum of all effects combined reduces to the sum of output effects.

2.3 Measure of Antitrust Harm

The net actual antitrust harm to total welfare is equal to the change in total profits in the chain, $\sum_{k=1}^{K} \triangle\pi_k$, plus the change in consumer surplus, $\triangle CS$. That is,

$$\triangle W = \sum_{k=1}^{g-1} \triangle\pi_k + \triangle\pi_g + \left[\sum_{k=g+1}^{K} \triangle\pi_k + \triangle CS\right] = d_U + \triangle\pi_g + d_D.$$

The *cartel gains* are $\triangle\pi_g$. The *downstream damages* $d_D = \sum_{k=g+1}^{K} \triangle\pi_k + \triangle CS$ correspond to losses in profits and consumer surplus by all direct and indirect purchasers. In addition, there are *upstream damages*, $d_U = \sum_{k=1}^{g-1} \triangle\pi_k$, equal to profit losses incurred by direct and indirect suppliers to the cartel.

We can use our decomposition of harm in equation (2) to evaluate each of these terms separately. We find

$$\triangle W = \sum_{k=1}^{K} (\xi_k - \omega_k + \sigma_k) + \xi_C + \sigma_C = \sum_{k=1}^{K} \sigma_k + \sigma_C,$$

where we used $\xi_1 = 0$ and the fact that the overcharge on layer $k+1$ equals the pass-on of layer k, $\xi_{k+1} = \omega_k$ for $k = 1, \ldots, K$ and $\xi_C = \omega_K$. The total welfare effect therefore coincides with the sum of the output effects.

Cartel profits are $\triangle \pi_g = \xi_g - \omega_g + \sigma_g$. Downstream harm can be represented as

$$d_D = \sum_{k=g+1}^{K} (\xi_k - \omega_k + \sigma_k) + \xi_C + \sigma_C = \xi_{g+1} + \sum_{k=g+1}^{K} \sigma_k + \sigma_C,$$

or the sum of all output effects of direct and indirect purchasers plus the direct purchaser overcharge. Upstream harm is equal to

$$d_U = \sum_{k=1}^{g-1} (\xi_k - \omega_k + \sigma_k) = -\omega_{g-1} + \sum_{k=1}^{g-1} \sigma_k = -\xi_g + \sum_{k=1}^{g-1} \sigma_k.$$

We are interested in the direct purchaser overcharge, ξ_{g+1}, in relation to these actual welfare effects. That is, we evaluate the ratio

$$\lambda_W = \frac{\triangle W}{\xi_{g+1}} = \lambda_g + \lambda_D + \lambda_U,$$

in which

$$\lambda_g = \frac{\triangle \pi_g}{\xi_{g+1}} = \frac{\xi_g + \sigma_g}{\omega_g} - 1, \tag{6}$$

are the cartel gains expressed in the direct purchaser overcharge,

$$\lambda_D = \frac{d_D}{\xi_{g+1}} = 1 + \sum_{k=g+1}^{K} \frac{\sigma_k}{\xi_{g+1}} + \frac{\sigma_C}{\xi_{g+1}}, \tag{7}$$

is the downstream harm to the direct purchaser overcharge ratio, and

$$\lambda_U = \frac{d_U}{\xi_{g+1}} = -\frac{\xi_g}{\omega_g} + \sum_{k=1}^{g-1} \frac{\sigma_k}{\xi_{g+1}}, \tag{8}$$

is that ratio upstream.

In addition to these aggregate measures, we specify the individual harm to direct purchasers and to consumers as:

$$\lambda_{g+1} = \frac{\xi_{g+1} - \omega_{g+1} + \sigma_{g+1}}{\xi_{g+1}} = 1 - \frac{\omega_{g+1} - \sigma_{g+1}}{\xi_{g+1}} \text{ and } \lambda_C = \frac{\xi_C + \sigma_C}{\xi_{g+1}}. \tag{9}$$

In the next section, we evaluate how these various ratios vary with the intensity of competition, the number of firms in each layer, the number of layers, and the position of the layer in which the anticompetitive behavior emerges in a specified vertical production model.

3 Quantifying Antitrust Damages using the Direct Purchaser Overcharge

In order to explicitly characterize the ratios introduced above, we need to further specify our model. Suppose that marginal costs are constant and identical for every firm in the same layer. That is, for layer k we have $c_{jk}(q) = c_k q$, for each $j \in \{1, \ldots, n_k\}$.[16] Let the inverse demand function be

$$P(Q) = a - bQ^{\gamma}, \tag{10}$$

with a, b and $\gamma > 0$.[17] Inverse demand is a convex function of quantity for $0 < \gamma < 1$, a concave function for $\gamma > 1$ and a linear function for $\gamma = 1$.

In this setup, the equilibrium quantity and prices can be expressed as follows:[18]

$$Q^* = \left[\frac{1}{b}\left(\prod_{i=1}^{K}\frac{n_i}{n_i+\gamma\vartheta_i}\right)\left(a-\sum_{j=1}^{K}c_j\right)\right]^{\frac{1}{\gamma}}, \tag{11}$$

$$p_k^* = \left(1-\prod_{i=1}^{k}\frac{n_i}{n_i+\gamma\vartheta_i}\right)\left(a-\sum_{j=1}^{K}c_j\right)+\sum_{l=1}^{k}c_l \qquad \forall\, k \in \{1,...,K\}. \tag{12}$$

The competitive benchmark is characterized by a vector of conjectural variations parameters $(\vartheta_1, \vartheta_2, \ldots, \vartheta_K) \in \times_{k=1}^{K}[0, n_k]$.

Collusion amongst the n_g firms in layer g (with $n_g > 1$) results in an increase in ϑ_g to $\vartheta_g^c \in (\vartheta_g, n_g]$. It follows straightforwardly from equation (11) that such an increase in any ϑ_k decreases the equilibrium quantity. For $n_g > 1$ and $\vartheta_g^c \in (\vartheta_g, n_g]$, we can write the ratio of collusive output to total competitive output r as

$$r = \frac{Q^g}{Q^*} = \left(\frac{n_g+\gamma\vartheta_g}{n_g+\gamma\vartheta_g^c}\right)^{\frac{1}{\gamma}}.$$

Note that, although both Q^* and Q^g depend on market characteristics of every layer, apart from γ, their ratio is a function only of characteristics of the colluding layer. If $\gamma = 1$, $r \in \left[\frac{1}{2}, 1\right)$, with the lower bound corresponding to perfect competition in layer g in the but-for world. An increase (decrease) in γ above (below) 1 decreases (increases) this lower bound value.[19]

Taken together, we can now characterize cartel profits in layer g in terms of the direct purchaser overcharge as

$$\lambda_g = \frac{\sigma_g}{\omega_g} - 1 = \frac{\gamma\vartheta_g}{n_g}\frac{(1-r)}{r(1-r^{\gamma})} - 1.$$

[16] In order to have gains from trade in this market, we naturally require $a > \sum_{j=1}^{K} c_j$. That is, the consumer's willingness to pay for the first unit (a) must exceed total costs to produce that unit ($\sum_{j=1}^{K} c_j$).

[17] Note that demand is nonnegative and nonincreasing as well for $a \geq 0$, $b < 0$ and $\gamma < 0$—see Genesove and Mullin (1998). We restrict attention to $b, \gamma > 0$, since in that case second-order conditions are always satisfied. Corbett and Karmarkar (2001) develop a multi-layered Cournot model with linear demand.

[18] See Proposition 4 in Appendix A.1.

[19] In particular, $\lim_{\gamma\to\infty} r = 1$ for all parameter values and $\lim_{\gamma\to 0} r = e^{-1}$ for $\vartheta_g^c = n_g$ and $\vartheta_g = 0$.

Note that $\lambda_g = -1$ if prior to collusion layer g was in perfect competition ($\vartheta_g = 0$ or $n_g \to \infty$). In that case, $\sigma_g = 0$ and the total cartel profit equals the overcharge on the direct purchasers. In all other benchmarks, (positive) cartel profits are always smaller than the direct purchaser overcharge. Next, we turn to the cartel effects down- and upstream.

3.1 Downstream Damages

Downstream from cartel layer g, it follows from equation (12) that equilibrium prices p_k^* (weakly) increase in all layers $k \geq g$, as each subsequent layer passes on part of the price increase it receives from its suppliers to its customers.[20] A full characterization of which player faces what passed-on price increase allows us to consider the direct purchaser overcharge ratios derived above. To begin with, consider

$$\lambda_{g+1} = \frac{\gamma\vartheta_{g+1}}{n_{g+1} + \gamma\vartheta_{g+1}} \frac{1 - r^{\gamma+1}}{r\left(1 - r^{\gamma}\right)}.$$

This expression immediately reveals that the direct purchaser overcharge must generally be a poor estimator for actual direct purchaser harm. In the case of linear demand, for example, we obtain $Q^* \leq 2Q^g$ from equation (11), resulting in $\lambda_{g+1} \in \left[0, \frac{3}{2}\right]$. The upperbound is reached when the pre-cartel equilibrium in layer g was perfectly competitive, so that $Q^* = 2Q^g$. The region only slightly changes when demand is non-linear.

The actual antitrust harm of direct purchasers will typically be small when there is strong competition between them, and/or competition in layer g was weak to begin with. In these cases, the direct purchaser overcharge will significantly overestimate the actual harm that direct purchasers suffer. In particular, if $n_{g+1} \geq 2$ and $\vartheta_{g+1} \leq 1$, in all cases $\lambda_{g+1} \leq 1$.[21] If on the other hand layer $g+1$ is governed by a monopolist or a cartel itself (*e.g.*, $\vartheta_{g+1} = n_{g+1}$), the direct purchaser overcharge always underestimates the actual harm sustained by indirect purchasers. Naturally, it is possible to construct market structures in which the direct purchaser overcharge turns out to be exact. This is so, for example, if direct purchasers have sufficient market power and pre-cartel competition in the colluding layer was strong. Such examples are non-generic, however.[22]

[20]For $k \geq g+1$, this *pass-on rate* R_k can be expressed as

$$R_k = \frac{\omega_k}{\xi_k} = \frac{p_k^g - p_k^*}{p_{k-1}^g - p_{k-1}^*} = \frac{n_k}{n_k + \gamma\vartheta_k}.$$

Note that unless in perfect competition ($\vartheta_k = 0$), each layer will absorb some of the price overcharge it receives. The less competitive a layer is, the lower is the pass-on fraction.

[21]Verboven and van Dijk (2007) propose their 'discounts' on the direct purchaser overcharge when awarding damages in direct purchaser suits on the claim that $\lambda_{g+1} \leq 1$, *i.e.*, that the pass-on effect would always outweigh the output effect, $\omega_{g+1} \geq \sigma_{g+1}$ and therefore the direct purchaser overcharge overestimates the actual harm. Note that this need not be true in our more general setting.

[22]Hellwig (2006) bases his argument for restricting standing to sue to direct purchasers to the claim that the direct purchaser overcharge exactly coincides with the actual harm if the direct purchaser layer is monopolized—and thus overestimates the actual harm in all other cases. Verboven

Next consider normalized harm to final consumers[23]

$$\lambda_C = \frac{\gamma}{\gamma+1}\frac{1-r^{\gamma+1}}{r\left(1-r^{\gamma}\right)}\prod_{i=g+1}^{K}\frac{n_i}{n_i+\gamma\vartheta_i}.$$

Again we find for $\gamma = 1$ that $\lambda_C \in \left[0, \frac{3}{2}\right]$ with a slightly changed upperbound for non-linear demand. If all intermediate layers downstream from the cartel are sufficiently competitive, the direct purchaser overcharge underestimates actual final consumer harm and $\lambda_C > 1$. If instead there is substantial market power in enough of these layers, the direct purchaser overcharge will overestimate consumer harm and $\lambda_C < 1$.

Aggregate downstream welfare effects relate to the direct purchaser overcharge as[24]

$$\lambda_D = \frac{1-r^{\gamma+1}}{r\left(1-r^{\gamma}\right)}\left(1-\frac{1}{\gamma+1}\prod_{i=g+1}^{K}\frac{n_i}{n_i+\gamma\vartheta_i}\right). \tag{13}$$

Clearly, λ_D decreases with r, a decrease in competition in one of the downstream layers, and an increase in the number of downstream layers. Note also that λ_D is unaffected by changes in the number of upstream layers and their competitiveness.

In the case of linear demand, $\frac{1}{2}\frac{Q^*+Q^g}{Q^g} \leq \lambda_D < \frac{Q^*+Q^g}{Q^g}$. So we find quite intuitively that downstream harm is greater when the cartel reduces output more, which is the case for example when the pre-cartel equilibrium is more competitive. When all intermediate layers are perfectly competitive, every layer fully passes on the overcharge which then is eventually borne by the end users, *i.e.*, $\lambda_D = \lambda_C = \frac{\xi_C+\sigma_C}{\xi_C} = \frac{1}{2}\frac{Q^*+Q^g}{Q^g} > 1$. This provides the lower bound of the actual harm. Monopolistic competition in the intermediate downstream layers increases actual downstream antitrust harm, with a strict upper bound of $\lambda_D < \frac{Q^*+Q^g}{Q^g}$.[25] Interestingly enough, the maximum limiting value of this strict upper bound is exactly 3, or the treble damages multiplier specified in Section 4 of the Clayton Act.[26]

The two top-panels of Figure 3 plot the value of absolute and relative aggregate downstream antitrust harm against the number of cartel members (upper-left panel) and the number of direct purchasers (upper-right panel) in an example with linear demand.[27] The value of λ_D increases monotonically with the number of firms in the

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and van Dijk (2007) reproduce this result as a special case of their analysis. The marginal price increases in both papers correspond to a marginal increase in ϑ_g in our setting, for which we obtain $\lambda_{g+1} = \frac{2\vartheta_{g+1}}{n_{g+1}+\vartheta_{g+1}}$, which indeed equals 1 for $\vartheta_{g+1} = n_{g+1}$—and is smaller otherwise.

[23] Note that λ_c can be written as $\lambda_c = \frac{1}{2}\frac{Q^*+Q^g}{Q^g}\mathbf{R}_C$, where $\mathbf{R}_C = \prod_{i=g+1}^{K} R_i = \frac{p_K^g - p_K^*}{p_g^\gamma - p_g^*}$ is the part of the price increase due to the cartel that ends up being paid by the final consumers.

[24] See Proposition 5 and Corollary 1 in Appendix A.1 for details of the derivation.

[25] This upper bound is reached with an infinite number of imperfectly competitive layers downstream. This limit case also implies zero equilibrium quantities, $\lim_{K\to\infty} Q^* = \lim_{K\to\infty} Q^g = 0$—see equation (11).

[26] Since the cartel reduces production, $Q^g < Q^*$. In addition, it follows from $\frac{Q^*}{Q^g} = \frac{n_g+\vartheta_g^c}{n_g+\vartheta_g}$ that $Q^* \leq 2Q^g$, with $Q^* = 2Q^g$ if the pre-cartel industry was perfectly competitive ($n_g \to \infty$ or $\vartheta_g = 0$) and the cartel sets the full cartel quantity ($\vartheta_g^c = n_g$). In this case $\frac{Q^*+Q^g}{Q^g} = 3$.

[27] Although the multiplier λ_D does not depend upon the values of a, b and $\sum_{k=1}^{K} c_k$, the direct purchaser overcharge ξ_{g+1} and the total downstream harm d_D do. We have chosen these parameters

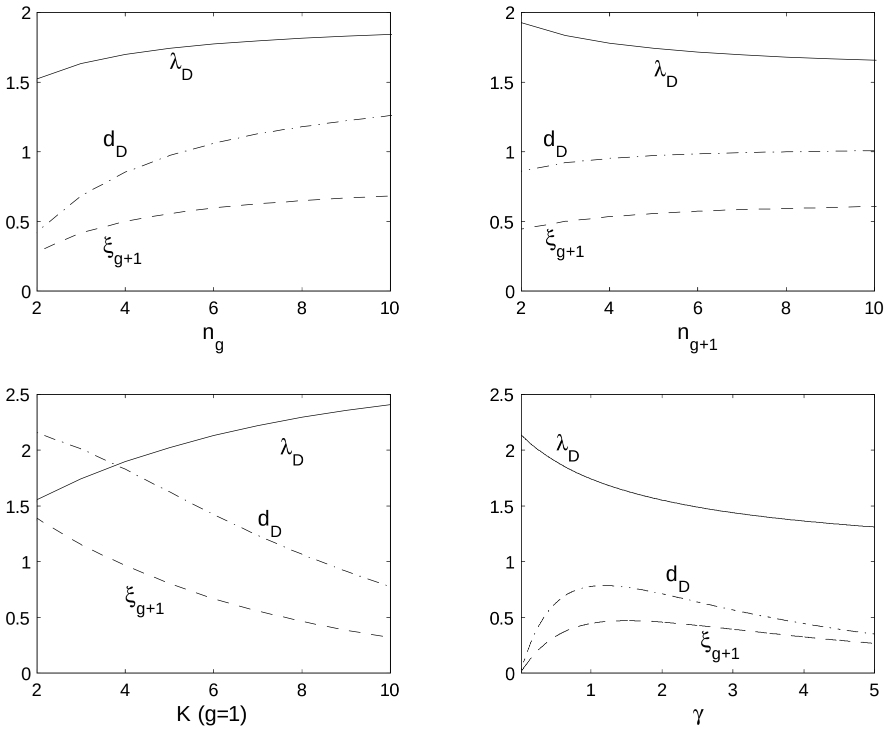

Figure 3: Downstream cartel effects under different numbers of firms per layer for $\gamma = 1$, $K = 5$, $n_k = 5$, $\vartheta_k = 1$, $g = 3$, and $\frac{1}{b}\left(a - \sum_{k=1}^{K} c_k\right)^2 = 10$. For the upper two panels, $\vartheta_3^c = 5$. The upper-left panel shows the effect of an increase in the number of cartel members, the upper-right panel in the number of direct purchasers. The lower-left panel displays downstream harm under different numbers of downstream layers of production when there is a cartel in the first layer for $\vartheta_1^c = 5$. The lower-right panel plots the effects of non-linearity of consumer demand.

cartel layer, since more competing firms implies a lower but-for price p_g^* and higher competitive output Q^*. These changes increase the direct purchaser overcharge ξ_{g+1}, as well as the output effects, $\sum_{k=g+1}^{K} \sigma_k + \sigma_C$. The net effect on the downstream damages measure d_D is positive, because output effects grow faster than the direct purchaser overcharge. The downstream multiplier decreases if competition in any downstream layer rises. Both the downstream harm d_D and the direct purchaser overcharge ξ_{g+1} increase because of increased output, but the latter increases at a faster rate. As a result, λ_D decreases in the number of direct purchasers.[28]

In the lower-left panel of Figure 3, the three parameters are plotted for the same specifications and a cartel in layer 1, varying the total number of downstream layers K.

in such a way that the three functions can be conveniently presented in the same graph, but they are otherwise non-specific.

[28] For this parameter configuration, the downstream damage multiplier is equal to $\frac{329}{216} \approx 1.52$ for $n_g = 2$ and equal to $\frac{47}{24} \approx 1.96$ when $n_g \to \infty$, as can be easily checked from (13). Moreover, it is equal to $\frac{52}{27} \approx 1.93$ for $n_{g+1} = 2$ and equal to $\frac{14}{9} \approx 1.56$ when $n_{g+1} \to \infty$

Increasing the number of similarly imperfectly competing downstream layers steeply decreases λ_D. Each additional downstream layer of production introduces an extra mark-up, which reduces implied demand for the colluding layer, thereby decreasing the direct purchaser overcharge. As a result, less demand is affected by the cartel.[29] The sum of output effects is the resultant of two opposite effects: the extra mark up increases downstream harm and the decrease in production decreases downstream harm. In the example in the figure, the first effect outweighs the second. However, the effect on the direct purchaser overcharge dominates, leading to a monotonic increase in λ_D.

The lower-right panel of Figure 3 shows the effect of variations in the shape of demand. Both d_D and ξ_{g+1} decrease both sides from linearity, resulting in a monotonically decreasing value of λ_D over the spectrum.

It appears in these examples that the ratio between actual downstream harm and the direct purchaser overcharge may be high. The following result establishes that it may indeed grow without bound.

Proposition 1 *For any finite number $\overline{M} > 0$, there exists a market structure such that $\lambda_D \geq \overline{M}$.*

The proposition shows that there is no upperbound on the downstream damages multiplier. This result obtains as long as there is some degree of market power in sufficiently many downstream layers.[30] The intuition is that in a two-layered model with cartelized suppliers directly supplying final consumers, the deadweight loss triangle becomes a smaller fraction of the overcharge quicker when demand is more concave than when demand is increasingly convex. In both limit demand curves, the output effect goes to zero, but a decrease in γ increases the output effect relative to the overcharge. The proof of Proposition 1 uses the fact that when demand becomes relatively elastic in the relevant region ($\gamma \to 0$), the output effect becomes large relative to the overcharge.

Finally, we compare total downstream harm with the sum total of direct purchaser overcharges and lost profits (dead-weight loss). That is, we define

$$\widetilde{\lambda}_D = \frac{d_D}{\xi_{g+1} + \sigma_{g+1}}.$$

For some parameter specifications, adding the direct purchasers' dead-weight loss to the denominator changes the damage multiplicator somewhat. The total direct effect remains a poor estimator of total true antitrust harm, however. To see this, first note that if layer $g+1$ is perfectly competitive—that is, $\vartheta_{g+1} \to 0$ and/or $n_{g+1} \to \infty$—there is no output effect for this layer of direct purchasers, *i.e.*, $\sigma_{g+1} = 0$. This

[29] Note that an increase in the number of downstream layers decreases the equilibrium quantity—as can be seen from equation (11)—but leaves the equilibrium price in layer g unaffected—see equation (12).

[30] In fact, λ_D is bounded from above by $(1 + K - g)(e - 1)$, with e being the natural number. This bound is exact for $\gamma \to 0$, $\vartheta_g = 0$ and $\vartheta_g^c = n_g$, and $\vartheta_k = n_k$ for all downstream layers $k = g+1, \ldots, K$. See Appendix A.2.

implies $\widetilde{\lambda}_D = \lambda_D$. Obviously, $\widetilde{\lambda}_D$ can therefore also take on any value. On the other hand, the output effect of the direct purchaser layer is large when this layer is fully monopolized, *i.e.*, $\vartheta_{g+1} = n_{g+1}$. Yet even in that case there exists no upper bound on $\widetilde{\lambda}_D$.[31] One way to interpret this finding is as the 'direct purchaser dead-weight loss' being a poor estimator of 'total chain deadweight losses'. Hence, even if lost profits would be recognized as (part of) antitrust harm, damage assessment at the direct purchaser level only in general provides no proper sense of the sum of actual downstream harm sustained in the chain.

3.2 Upstream Damages: Output Effects

Upstream from the cartel, the derived demand for inputs is reduced. Depending on market conditions, suppliers may optimally respond to this shift in demand by in- or decreasing output, resulting in an ambiguous effect on upstream prices. Under constant marginal costs of production, in our chosen demand specification (10) the price decreasing effect from reduced demand and the price increasing effect from reduced production exactly offset. To see that upstream prices are indeed not affected by downstream overcharges, note that in equation (12) the equilibrium price in layer k does not depend on ϑ_l or n_l for any $l > k$. This implies that collusion in layer g will not have any effect on prices upstream: $p_k^g = p_k^*$ for all $k < g$ and all upstream overcharges and pass-on effects vanish, $\xi_k = 0$ for $k = 1, \ldots, g$ and $\omega_k = 0$ for $k = 1, \ldots, g-1$.[32] This setup allows us to first focus exclusively on upstream output effects, which also suffice for our main results. In the next subsection, we offer an example of an upstream input price decrease by which damages are passed up.

Moreover, the case of linear demand suffices to establish some limit properties of λ_U. Aggregate upstream antitrust harm relates to the direct purchaser overcharge as

$$\lambda_U = \frac{(1-r)}{r\,(1-r^\gamma)} \frac{n_g + \gamma\vartheta_g}{n_g} \left(\prod_{i=1}^{g-1} \frac{n_i + \gamma\vartheta_i}{n_i} - 1 \right), \tag{14}$$

so that we immediately have the following result.

Proposition 2 *For any finite number $\overline{M} > 0$, there exists a market structure such that $\lambda_U \geq \overline{M}$.*

Note also that λ_U increases with a decrease in competition in one of the upstream layers, as well as with an increase in the number of imperfectly competitive upstream layers. It is furthermore invariant to changes in the number of downstream layers and their competitiveness.

The upper-left panel of Figure 4 shows numerically that both upstream antitrust harm d_U and the direct purchaser overcharge ξ_{g+1} grow with the number of firms

[31] See Lemma 3 in Appendix A.2.

[32] Greenhut and Ohta (1976) and Haring and Kaserman (1978) find analogously for the case of linear demand and constant marginal costs that output changes but upstream input prices remain the same under vertical integration. In the next subsection we discuss variations in demand and cost functions.

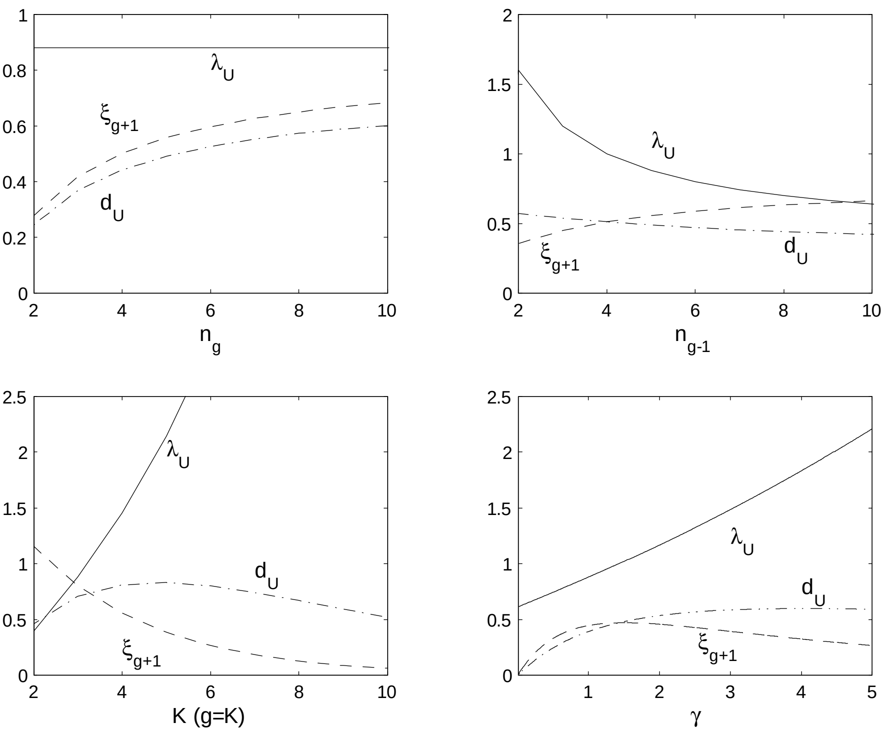

Figure 4: Upstream cartel effects under different numbers of firms per layer for $\gamma = 1$, $K = 5$, $n_k = 5$, $\vartheta_k = 1$, $g = 3$ and $\frac{1}{b}\left(a - \sum_{k=1}^{K} c_k\right)^2 = 10$. For the upper two panels, $\vartheta_3^c = 5$. The upper-left panel shows the effect of an increase in the number of cartel members, the upper-right panel in the number of direct suppliers. The lower-left panel displays upstream harm under different numbers of downstream layers of production when there is a cartel in the final layer K for $\vartheta_K^c = 5$. The lower-right panel displays the effects of non-linearity of consumer demand.

in the colluding layer—increasing competition in the 'but-for' world—but that the upstream damage multiplier λ_U remains constant. The upper-right panel shows the effect of an increase in the number of firms in one of the upstream layers. This decreases upstream harm d_U, since an increase in the number of firms in one layer decreases the mark up, and thereby the output effect, in that layer. This effect is only partially outweighed by an increase in the output effect that occurs due to the increase in production. At the same time, the direct purchaser overcharge decreases with an increase in the number of firms in an upstream layer and the net effect is obviously a decrease in the upstream damage multiplier.[33]

The lower-left panel shows how the upstream damage multiplier varies with the number of upstream layers. If the number of (imperfectly competitive) upstream layers increases, production decreases, which decreases the direct purchaser overcharge.

[33]For the parameter values here considered, the upstream damage multiplier is equal to $\frac{8}{5}$ for $n_{g-1} = 2$ and equal to $\frac{2}{5}$ when $n_{g-1} \to \infty$, as can be easily checked from equation (14).

The relationship between downstream damages d_U and the number of upstream layers is nonmonotonic in this linear example. An increase in the number of layers introduces additional mark-ups but decreases production. The first effect has a positive, and the second a negative impact upon upstream output effects. From Figure 4 it follows that the first (second) effect dominates when the number of layers is small (large). The net effect of an increase of the number of upstream layers on the upstream damage multiplier is always positive.

3.3 Upstream Damages: Price Effects

Variations in demand or the cost of production can generate upstream price effects in both directions. Input prices may increase when the cartel reduces demand. Direct suppliers may also obtain a lower price for their inputs by the cartel members than they would under competition, however, and so face an 'undercharge'. Figure 5 illustrates an example of input prices in layer $g-1$ decreasing as a result of a cartel forming in layer g when the marginal upstream costs of production increase in production.

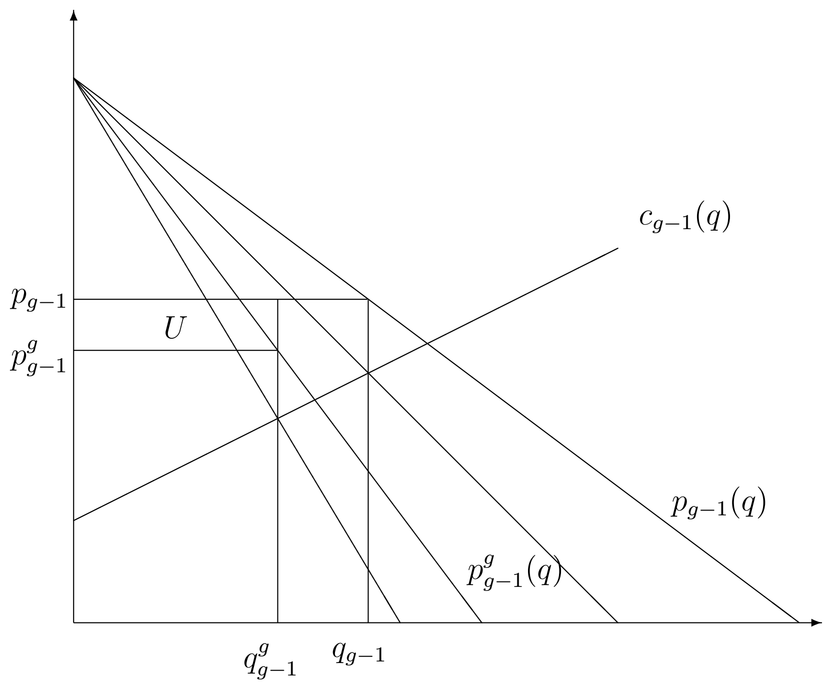

Figure 5: Direct sellers undercharged by a purchasers cartel.

The derived demand for inputs under downstream competition, $p_{g-1}(q)$, turns inwards to $p^g_{g-1}(q)$. Profit maximization given $c_{g-1}(q)$ results in lower input prices to the purchaser cartel, $p^g_{g-1} < p_{g-1}$. As a result, the upstream industry sustains an undercharge of size $\left(p_{g-1} - p^g_{g-1}\right) q^g_{g-1}$ on its actual sales—or area U in the figure.

Given linear demand, upstream prices increase after downstream collusion when costs are concave, and decrease when costs are convex.[34] The intuition for the latter is straightforward from the case of perfect competition in the upstream market, in which prices are equal to marginal costs. Decreasing returns to scale result in lower marginal costs of production in equilibrium when the quantity of inputs demanded is reduced. It carries over to other forms of imperfect competition upstream that our model allows. Hence, direct suppliers that operate under decreasing returns to scale can have a positive antitrust damage claim even if the method of quantifying upstream harm would be restricted to the mirror image of the overcharge method and deadweight-losses were not awarded.

3.4 Cartel Location and the Distribution of Harm

Equations (13) and (14), as well as Figure 4 suggest that the upstream damages multiplier λ_U is substantially larger than the downstream damage multiplier λ_D. This raises the question whether it is possible for total antitrust harm to be higher when the location of the colluding layer is closer to the final consumers. A priori such proximity might increase the harm done upstream more than it decreases the harm done downstream. While indeed the multipliers do not trade off perfectly, the absolute level of total cartel harm is unaffected by the location of the cartel.

Proposition 3 *Consider two distinct layers, g and $h > g$ that have the same characteristics: $n_h = n_g$, $\vartheta_h = \vartheta_g$, and, if one of these layers would collude, $\vartheta_h^c = \vartheta_g^c$. Let $\triangle W_k$ be the change in welfare if layer k colludes. Then $\triangle W_h = \triangle W_g$, $\xi_{h+1} < \xi_{g+1}$ and $\lambda_{W,h} > \lambda_{W,g}$.*

All other things equal, the lower the cartel is in the chain, the smaller is the direct purchaser overcharge, while total cartel harm remains the same. Therefore, the upstream damages multiplier increases when the cartel is lower in the chain. The reason for this is that if the number of layers upstream from where the cartel forms is large, each of these layers will have put some mark-up on the price. This effectively

[34] Analytical results quickly become intractable in longer supply chains. From tedious but straightforward computations it follows that in a chain with two layers of production the upstream Cournot equilibrium price decreases with an increase in ϑ_2, whenever

$$(\Psi_P + \Psi_c) P'(Q) >$$
$$\left((n_2 + \vartheta_2) P'(Q) + \vartheta_2 Q P''(Q) - c_2''\left(\frac{Q}{n_2}\right)\right)((n_1+1) P'(Q) + QP''(Q)).$$

where $\Psi_P = (n_1+1)(n_2+\vartheta_2) P'(Q) + (n_2 + (n_1+3)\vartheta_2) QP''(Q) + \vartheta_2 Q^2 P'''(Q)$ and $\Psi_c = -n_2 c_1''\left(\frac{Q}{n_1}\right) - (n_1+1) c_2''\left(\frac{Q}{n_2}\right) - \frac{Q}{n_2} c_2'''\left(\frac{Q}{n_2}\right)$. If consumer demand is linear and marginal costs downstream are constant, this condition reduces to $c_1''\left(\frac{Q}{n_1}\right) > 0$. Furthermore, the class of demand specifications for which there are no upstream price effects when the marginal costs of production are constant both up- and downstream is characterized by

$$P''(Q) P'(Q) + Q\left\{P'''(Q) P'(Q) - [P''(Q)]^2\right\} = 0.$$

Obviously, whereas (10) does, many nonlinear demand functions do not satisfy this condition.

reduces the scope for abuse of market power by the colluding layer. At the same time, more layers upstream and less downstream from the cartel implies that the direct purchaser overcharge misses more of the total of effects.

Formally, the price overcharge is given as

$$p_g^g - p_g^* = \Psi \prod_{i=1}^{g} \frac{n_i}{n_i + \gamma\vartheta_i},$$

where Ψ is a function of the parameters of the model and independent of the location of the colluding layer. Clearly, when the number of imperfectly competitive layers—or for that matter market power of existing upstream layers—increases, this difference decreases. As a consequence, the total welfare measure λ_W increases when the colluding layer is further downstream in the chain of production, not because absolute welfare effects are higher, but because the direct purchaser overcharge goes down. Figure 6 illustrates.

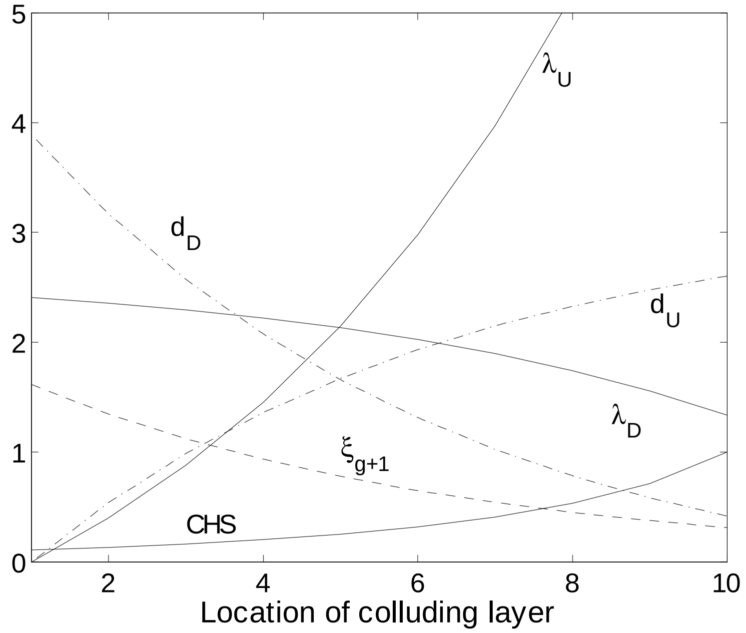

Figure 6: Down- and upstream damages and the CHS as a function of the location of the cartel in the chain of production for $n_k = 5$ and $\vartheta_k = 1$ for all k, $\frac{1}{b}\left(a - \sum_{k=1}^{K} c_k\right)^2 = 50$ and $\vartheta_g^c = 5$.

Given that final output is independent of the location of the cartel, so is the increase of final consumer prices. Therefore if the colluding layer is higher up in the chain of production, it has to increase prices by more, in order to achieve the same increase in final consumer prices. This is because part of the price increase is absorbed by intermediate downstream layers. For this reason, the direct purchaser overcharge for a cartel higher up the chain is higher. Hence, the closer the cartel is to the final customers, the more the direct purchaser overcharge underestimates the actual harm, the larger share of which is borne by the producers upstream in the chain of production.

Figure 6 also plots the 'consumer harm share' (CHS), a concept introduced in Boone and Mueller (2008) as the share of final consumers in the sum total of downstream harm, that is $CHS = \frac{\Delta CS}{d_D}$. So defined, the value of CHS increases, the closer the cartel is to the consumer market. In our setup, the CHS is more naturally defined as the share in the total welfare effects. Since the sum of all welfare effects in the chain is independent of the location of the cartel, in this definition the consumer harm share is constant.

4 Concluding Remarks

We have assessed the pass-on of antitrust welfare effects in longer vertical supply chains, in which a cartel may form in any layer of production. We find that the direct purchaser overcharge generally grossly underestimates the total antitrust harm. While the direct purchaser overcharge is equal to the sum of all passed on overcharges downstream, it misses the output effects in every layer. Including the direct purchaser output effect in the damage assessment does not generally remedy this. The share in total harm sustained by suppliers to the cartel may be large. Relying on the direct purchaser overcharge becomes increasingly problematic, the longer the vertical chain of production and the closer the collusion occurs to the final consumers.

We conclude that the established overcharge method is only suitable in a small class of antitrust damages cases. There exist no simple multiplication factors to correct the direct purchaser overcharge for actual markets that rely on basic market structure characteristics. Nor are the lost profits of the direct purchasers a good measure for total chain deadweight losses. This implies that it will not generally suffice to collect the direct purchaser overcharge first, and then redistribute this money over all cartel victims later.

In light of our results, it appears to be an unbalanced spending of resources to devote considerable effort and econometric expertise to the characterization of but-for worlds, as is often the case in U.S. antitrust damage cases, only to subsequently use the so found but-for price to calculate the direct purchaser overcharge. With the information obtained in such detailed but-for economic analyses, structural estimations are often possible and the courts would do well to consider them. The same is true for claims of antitrust injury sustained by direct and indirect suppliers to a cartel.[35]

We warn in this context also against the European Commission's call for "simplified rules on estimating the loss" from infringements of the competition rules in its 2008 White Paper [36] The Commission's stated primary objective and guiding principle in this document is full compensation of victims of a breach of the EC antitrust rules—which is in line with relevant decisions of the European Court of Justice. It furthermore recognizes "actual damages, lost profits, and interest" for direct as well

[35] A creative suggestion for a practical and efficient method to quantify and apportion antitrust damages is given in Rüggeberg and Schinkel (2006).

[36] Commission of the European Communities (2008), p.3 and p.7.

as indirect purchasers. The Commission should realize that there is no hope for very simple rules on antitrust damage estimation.[37]

We have further clarified the antitrust damages that cartels cause to their direct and indirect suppliers. Analogous to customers of a cartel facing higher prices and reduced quantities of output, suppliers to the cartel may obtain lower prices for a reduced volume of input sales. Section 4 of the Clayton Acts states that:

> "Any person who shall be injured in his business or property by reason of anything forbidden in the antitrust laws [...] shall recover threefold the damages by him sustained ... "

The so-called 'target-area' concept of antitrust damages recognizes a "zone of harm" caused by the violation, for which the Supreme Court has suggested a broad interpretation of Section 4 in *Radovic* (1957) that also includes suppliers.[38] In *Wilson v. Ringsby Truck Lines* (1970), truck drivers and warehousemen of Ringsby, a common carrier transporting between Colorado and Wyoming, were indeed recognized as antitrust victims when their services were no longer needed after Ringsby joined a cartel that divided their markets geographically.[39]

In *Associated Contractors v. Carpenters* (1983), however, the Supreme Court reinterpreted the meaning of Section 4 narrowly and against its own opinions in *Mandeville Island Farms v. Sugar* (1948) and *Radovic* (1957), in part on the understanding that the intent of Congress in passing the Sherman Act had been to create an effective remedy primarily for consumers.[40] In addition to this, the Court found the chain of causation between the upstream injury and the downstream cartel "somewhat vaguely defined" and therefore "indirect".[41] It appears that the upstream effects of a downstream cartel are not properly viewed as antitrust damages in U.S. antitrust law. It remains an open question of law where—in both directions: up- and downstream—in antitrust cases are the points beyond which the wrongdoers should no longer be held liable.

Even though we offer a more general picture of antitrust effects than the existing the literature, ours also remains a partial model. We have abstracted from nonlinear cartel price elements, which can reduce deadweight-losses, but also from additional negative effects that many cartels have on product quality and innovation efforts. We have also not considered the effects of partial vertical integration in the chain of production. The cost structures we use are special—albeit that our results remain qualitatively unchanged in various variations with smoothly in- and decreasing marginal costs of production. In addition, we have assumed that cartels do not use their

[37] In addition, the Commission means to assist indirect purchasers by introducing a peculiar "rebuttable presumption that the illegal overcharge was passed on to them in its entirety." Commission of the European Communities (2008), p. 8. The use of "them" in this quote suggests that the drafters of the White Paper indeed had the mainstream two-layer upstream cartel model in mind.

[38] *Radovich v. National Football League* 352 U.S. 445 (1957). See Comment (1972), pp.403-4.

[39] See *Wilson v. Ringsby Truck Lines, Inc.*, 320 F. Supp. 699 (D. Colo. 1970).

[40] *Associated Contractors v. Carpenters* (1983), 529-535.

[41] *Associated Contractors v. Carpenters* (1983), pp.540-543.

market power *vis-a-vis* suppliers, which they probably could do profitably.[42]

Our assumptions of homogeneity of products within each layer and one-to-one production between layers are not entirely innocuous either. Our fixed-proportion production technology implies that products of different layers are perfect complements. When there would be substitution possibilities between inputs instead, a price increase in one layer might induce firms in the next layer to substitute away from that input and towards an input produced in another chain all together. We ignore this competitive constraint on pricing in our analysis. The industry supplying the other input is furthermore likely to benefit from the cartel, since its demand may go up and it can raise prices as a result of the reduced competition—this is sometimes called the 'umbrella effect' of antitrust violations.[43]

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[43] See Areeda and Hovenkamp (1992), para. 337.3 on the rules of standing for "umbrella" plaintiffs. The authors argue that purchasers from non-cartel members that were overcharged because of the umbrella effect should generally be granted standing to sue the cartel for damages, in particular when products are "homogeneous". When products are differentiated, the authors recognize that it may be difficult to draw the line where standing ends.

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A Characterizations and Proofs

In this appendix we present the analysis that underlies the text.

A.1 Intermediate Results

Our first intermediate result characterizes the implied inverse demand function for each layer k.

Lemma 1 *Given final consumer demand $P(Q) = a - bQ^{\gamma}$, constant marginal costs c_k in layer k and conjectural variations parameter ϑ_k in layer k, the implied inverse demand function faced by firms in layer k is*

$$p_k(Q) = a - \left(b \prod_{i=k+1}^{K} \frac{n_i + \gamma\vartheta_i}{n_i} Q^{\gamma} + \sum_{l=k+1}^{K} c_l \right), \tag{15}$$

for $k = 1, \ldots, K-1$. Furthermore, $p_K(Q) = P(Q)$.

Proof. We will show that for a given K, (15) holds for all $1 \leq k \leq K-1$. Consider firm i in layer k. It maximizes profits $p_k(Q)\, q_i - (c_k - p_{k-1})\, q_i$, where $p_k(Q)$ is the implied inverse demand function that the industry in layer k faces. Using symmetry ($q_i = Q/n_k$) and conjectural variations ($\vartheta_k = dQ/dq_i$) the first-order condition equals (1). Solving for p_{k-1} then gives

$$p_{k-1}(Q) = p_k(Q) + \frac{\vartheta_k}{n_k} Q p_k'(Q) - c_k. \tag{16}$$

Given $p_k(Q)$, the implied inverse demand for layer $k-1$, $p_{k-1}(Q)$ can therefore be determined recursively. We need to show that (15) satisfies the recursive relation (16) for all $k = 2, \ldots, K$. First consider $k = K$. Using $p_K(Q) = P(Q) = a - bQ^{\gamma}$ (16) reduces to $p_{K-1}(Q) = a - b\frac{n_K + \gamma\vartheta_K}{n_K} Q^{\gamma} - c_K$, which is equivalent to (15) for $k = K-1$. Now we proceed by induction. Assuming that (15) holds for k we want to show that it also holds for $k-1$. We have

$$\begin{aligned}
p_{k-1}(Q) &= p_k(Q) + p_k'(Q) \frac{\vartheta_k}{n_k} Q - c_k \\
&= a - \left(b \prod_{i=k+1}^{K} \frac{n_i + \gamma\vartheta_i}{n_i} Q^{\gamma} + \sum_{l=k+1}^{K} c_l \right) - \left(\gamma b \prod_{i=k+1}^{K} \frac{n_i + \gamma\vartheta_i}{n_i} \right) Q^{\gamma} \frac{\vartheta_k}{n_k} - c_k \\
&= a - \left(b \frac{n_k + \gamma\vartheta_k}{n_k} \prod_{i=k+1}^{K} \frac{n_i + \gamma\vartheta_i}{n_i} Q^{\gamma} + c_k + \sum_{l=k+1}^{K} c_l \right) \\
&= a - \left(b \prod_{i=k}^{K} \frac{n_i + \gamma\vartheta_i}{n_i} Q^{\gamma} + \sum_{l=k}^{K} c_l \right).
\end{aligned}$$

Therefore equation (15) holds for all k with $1 \leq k \leq K-1$. Finally, it is easily checked that $\gamma > 0$ is a sufficient condition for the second-order condition for an optimum to be satisfied for every individual firm at the equilibrium. ■

Knowing the implied inverse demand functions (15), we can now determine equilibrium quantities and prices.

Proposition 4 *In this model, equilibrium prices and quantities are given as*

$$Q^* = \left[\frac{1}{b}\left(\prod_{i=1}^{K}\frac{n_i}{n_i+\gamma\vartheta_i}\right)\left(a-\sum_{j=1}^{K}c_j\right)\right]^{\frac{1}{\gamma}}, \tag{17}$$

$$p_k^* = \left(1-\prod_{i=1}^{k}\frac{n_i}{n_i+\gamma\vartheta_i}\right)\left(a-\sum_{j=1}^{K}c_j\right)+\sum_{l=1}^{k}c_l \qquad \forall\, k\in\{1,\ldots,K\}. \tag{18}$$

Aggregate profits of firms in layer k and consumer surplus are then respectively given by

$$\pi_k^* = \left(\frac{1}{b}\right)^{\frac{1}{\gamma}}\frac{\gamma\vartheta_k}{n_k+\gamma\vartheta_k}\prod_{i=1}^{k-1}\frac{n_i}{n_i+\gamma\vartheta_i}\left(\prod_{i=1}^{K}\frac{n_i}{n_i+\gamma\vartheta_i}\right)^{\frac{1}{\gamma}}\left(a-\sum_{j=1}^{K}c_j\right)^{\frac{\gamma+1}{\gamma}} \tag{19}$$

$$CS = \frac{\gamma}{\gamma+1}\left(\frac{1}{b}\right)^{\frac{1}{\gamma}}\left(\prod_{i=1}^{K}\frac{n_i}{n_i+\gamma\vartheta_i}\right)^{\frac{\gamma+1}{\gamma}}\left(a-\sum_{j=1}^{K}c_j\right)^{\frac{\gamma+1}{\gamma}}. \tag{20}$$

Proof. Using (15) the first-order condition (1) for $k=1$ reduces to

$$a-\left(b\prod_{i=1}^{K}\frac{n_i+\gamma\vartheta_i}{n_i}Q^{\gamma}+\sum_{l=1}^{K}c_l\right)=0.$$

Solving for Q gives (17). Substituting Q^* into (15) gives (18). Profits and consumer surplus follow from substituting (17) and (18) in $\pi_k^* = \left(p_k^*-p_{k-1}^*-c_k\right)Q^*$ and $CS = \int_0^{Q^*}P(Q)\,dQ - p_K^*Q^* = \frac{\gamma}{\gamma+1}b\,(Q^*)^{\gamma+1} = \frac{\gamma}{\gamma+1}\left(a-p_K^*\right)Q^*$, respectively. ∎

We are interested in how the equilibrium quantity Q^* and equilibrium prices p_k^* depend upon the underlying model parameters. It is easily checked that Q^* increases with an increase in a or n_i and with a decrease in ϑ_i, b or the number of (imperfectly competitive) layers K. The equilibrium price for layer k, p_k^*, increases with an increase in a, γ, c_i or ϑ_i, for $i\leq k$, and with a decrease in n_i for $i\leq k$, with a decrease in c_i for $l>k$.

The dependence of Q^* on γ, however, is ambiguous and Q^* could either decrease or increase with γ.[44] Moreover, we have

$$\lim_{\gamma\to\infty}p_k^* = a-\sum_{j=k+1}^{K}c_j \text{ and } \lim_{\gamma\to 0}p_k^* = \sum_{j=1}^{k}c_j.$$

Firms in layer 1 have all the market power, that is, they extract the entire surplus by setting price p_1^* equal to a minus the marginal costs of the other layers. The other layers price competitively, in the sense that they only recover their marginal costs.

[44] Take for example the case with $\vartheta_i = 0$ for all i. Then Q^* increases (decreases) with γ if $a-\sum_{k=1}^{K}c_k$ is larger (smaller) than b.

Using $\lim_{\gamma\to\infty}\left(1+\frac{\vartheta_i}{n_i}\gamma\right)^{\frac{1}{\gamma}}=1$ and $\lim_{\gamma\to 0}\left(1+\frac{\vartheta_i}{n_i}\gamma\right)^{\frac{1}{\gamma}}=\exp\left[\frac{\vartheta_i}{n_i}\right]$, respectively, we find

$$\lim_{\gamma\to\infty} Q^*=1 \text{ and } \lim_{\gamma\to 0} Q^*=\begin{cases} 0, & \text{if } b>a-\sum_{j=1}^{K} c_j \\ \exp\left[-\sum_{i=1}^{K}\frac{\vartheta_i}{n_i}\right], & \text{if } b=a-\sum_{j=1}^{K} c_j \\ \infty, & \text{if } b<a-\sum_{j=1}^{K} c_j \end{cases}.$$

Hence, when demand becomes infinitely concave ($\gamma\to\infty$), the equilibrium quantity is 1, independent of all other parameters. Therefore, when demand becomes infinitely concave $(\gamma\to 0)$, the equilibrium price is 0 for all layers, independent of all other parameters. The equilibrium quantity is 0 (∞) when $b>(<)\,a-\sum_{j=1}^{K} c_j$; only when this condition holds with equality equilibrium quantity is positive and finite.

Collusion in layer g is modelled as an increase in the conjectural variations parameter from ϑ_g to $\vartheta_g^c\in(\vartheta_g,n_g]$. Cartel quantity and prices are

$$Q^g = \left[\frac{1}{b}\frac{n_g+\gamma\vartheta_g}{n_g+\gamma\vartheta_g^c}\left(\prod_{i=1}^{K}\frac{n_i}{n_i+\gamma\vartheta_i}\right)\left(a-\sum_{j=1}^{K}c_j\right)\right]^{\frac{1}{\gamma}}, \tag{21}$$

$$p_k^g = \begin{cases}\left(1-\frac{n_g+\gamma\vartheta_g}{n_g+\gamma\vartheta_g^c}\prod_{i=1}^{k}\frac{n_i}{n_i+\gamma\vartheta_i}\right)\left(a-\sum_{j=1}^{K}c_j\right)+\sum_{l=1}^{k}c_l & k\geq g \\ p_k^* & k<g\end{cases}. \tag{22}$$

The next lemma provides expressions for pass-on and output effects (recall that $\xi_{k+1}=\omega_k$).

Lemma 2 *The pass-on effect ω_k for layer $k\geq g$ is given by*

$$\begin{aligned}\omega_k &= \left(\frac{1}{b}\right)^{\frac{1}{\gamma}}\left(\frac{n_g+\gamma\vartheta_g}{n_g+\gamma\vartheta_g^c}\right)^{\frac{1}{\gamma}}\left(\frac{\gamma\vartheta_g^c-\gamma\vartheta_g}{n_g+\gamma\vartheta_g^c}\right)\left(\prod_{i=1}^{k}\frac{n_i}{n_i+\gamma\vartheta_i}\right) \\ &\quad\times\left(\prod_{i=1}^{K}\frac{n_i}{n_i+\gamma\vartheta_i}\right)^{\frac{1}{\gamma}}\left(a-\sum_{j=1}^{K}c_j\right)^{\frac{\gamma+1}{\gamma}},\end{aligned} \tag{23}$$

and $\omega_k=0$ for every $k\leq g-1$. The output effect for layer k and the output effect for consumers are given as

$$\begin{aligned}\sigma_k &= \left(\frac{1}{b}\right)^{\frac{1}{\gamma}}\frac{\gamma\vartheta_k}{n_k}\left(1-\left(\frac{n_g+\gamma\vartheta_g}{n_g+\gamma\vartheta_g^c}\right)^{\frac{1}{\gamma}}\right)\left(\prod_{i=1}^{k}\frac{n_i}{n_i+\gamma\vartheta_i}\right) \\ &\quad\times\left(\prod_{i=1}^{K}\frac{n_i}{n_i+\gamma\vartheta_i}\right)^{\frac{1}{\gamma}}\left(a-\sum_{j=1}^{K}c_j\right)^{\frac{\gamma+1}{\gamma}},\end{aligned} \tag{24}$$

$$\begin{aligned}\sigma_C &= \left(\frac{1}{b}\right)^{\frac{1}{\gamma}}\left(\frac{\gamma+\left(\frac{n_g+\gamma\vartheta_g}{n_g+\gamma\vartheta_g^c}\right)^{\frac{\gamma+1}{\gamma}}}{\gamma+1}-\left(\frac{n_g+\gamma\vartheta_g}{n_g+\gamma\vartheta_g^c}\right)^{\frac{1}{\gamma}}\right) \\ &\quad\times\left(\prod_{i=1}^{K}\frac{n_i}{n_i+\gamma\vartheta_i}\right)^{\frac{\gamma+1}{\gamma}}\left(a-\sum_{j=1}^{K}c_j\right)^{\frac{\gamma+1}{\gamma}}.\end{aligned} \tag{25}$$

Proof. Straightforward computations show that (for $k \geq g$) we have

$$\begin{aligned}
p_k^g - p_k^* &= \frac{\gamma\vartheta_g^c - \gamma\vartheta_g}{n_g + \gamma\vartheta_g^c} \left(\prod_{i=1}^{k} \frac{n_i}{n_i + \gamma\vartheta_i} \right) \left(a - \sum_{j=1}^{K} c_j \right), \\
p_k^* - p_{k-1}^* - c_k &= \frac{\gamma\vartheta_k}{n_k} \left(\prod_{i=1}^{k} \frac{n_i}{n_i + \gamma\vartheta_i} \right) \left(a - \sum_{j=1}^{K} c_j \right), \\
Q^* - Q^g &= \left(1 - \left(\frac{n_g + \gamma\vartheta_g}{n_g + \gamma\vartheta_g^c} \right)^{\frac{1}{\gamma}} \right) \left[\frac{1}{b} \left(\prod_{i=1}^{K} \frac{n_i}{n_i + \gamma\vartheta_i} \right) \left(a - \sum_{j=1}^{K} c_j \right) \right]^{\frac{1}{\gamma}}.
\end{aligned}$$

Equations (23)–(24) follow immediately from substituting the above expressions into $\omega_k = Q^g \left(p_k^g - p_k^* \right)$ and $\sigma_k = (Q^* - Q^g) \left(p_k^* - p_{k-1}^* - c_k \right)$. Furthermore, we have

$$\sigma_C = \int_{Q^g}^{Q^*} \left[P(Q) - P(Q^*) \right] dQ = \frac{1}{\gamma + 1} b \left[(Q^g)^{\gamma+1} - (Q^*)^{\gamma+1} \right] + b (Q^*)^{\gamma} (Q^* - Q^g).$$

Equation (25) then follows from substituting $Q^g = rQ^*$, with $r = \left(\frac{n_g + \gamma\vartheta_g}{n_g + \gamma\vartheta_g^c} \right)^{\frac{1}{\gamma}}$. ■

Using Lemma 2, we can express the measures of harm discussed in Section 2.3 in terms of the parameters of the model.

Proposition 5 *Denote by* $r = \frac{Q^g}{Q^*} = \left(\frac{n_g + \gamma\vartheta_g}{n_g + \gamma\vartheta_g^c} \right)^{\frac{1}{\gamma}}$ *the fraction by which the cartel reduces output. The damages measures (7)–(9) are equal to*

$$\begin{aligned}
\lambda_D &= \frac{1 - r^{\gamma+1}}{r (1 - r^{\gamma})} \left(1 - \frac{1}{\gamma + 1} \prod_{i=g+1}^{K} \frac{n_i}{n_i + \gamma\vartheta_i} \right), && (26) \\
\lambda_U &= \frac{(1 - r)}{r (1 - r^{\gamma})} \frac{n_g + \gamma\vartheta_g}{n_g} \left(\prod_{i=1}^{g-1} \frac{n_i + \gamma\vartheta_i}{n_i} - 1 \right), && (27) \\
\lambda_g &= \frac{\gamma\vartheta_g}{n_g} \frac{(1 - r)}{r (1 - r^{\gamma})} - 1, && (28) \\
\lambda_{g+1} &= \frac{\gamma\vartheta_{g+1}}{n_{g+1} + \gamma\vartheta_{g+1}} \frac{1 - r^{\gamma+1}}{r (1 - r^{\gamma})}, \text{ and} && (29) \\
\lambda_C &= \frac{\gamma \left(1 - r^{\gamma+1} \right)}{(\gamma + 1) r (1 - r^{\gamma})} \prod_{i=g+1}^{K} \frac{n_i}{n_i + \gamma\vartheta_i}. && (30) \\
\lambda_W &= \left(\frac{n_g + \gamma\vartheta_g^c}{\gamma\vartheta_g^c - \gamma\vartheta_g} \right) \left[\frac{r^{\gamma} - r^{-1}}{\gamma + 1} \left(\prod_{i=g+1}^{K} \frac{n_i}{n_i + \gamma\vartheta_i} \right) + \left(r^{-1} - 1 \right) \left(\prod_{i=1}^{g} \frac{n_i + \gamma\vartheta_i}{n_i} \right) \right] && (31)
\end{aligned}$$

Proof. Substituting equations (23)–(25) into (7) and using $\left(\frac{n_g + \gamma\vartheta_g}{n_g + \gamma\vartheta_g^c} \right)^{\frac{1}{\gamma}} = r$ and $\frac{\gamma\vartheta_g^c - \gamma\vartheta_g}{n_g + \gamma\vartheta_g^c} = 1 - r^{\gamma}$ we obtain

$$\lambda_D = 1 + \frac{1 - r}{r (1 - r^{\gamma})} \sum_{k=g+1}^{K} \frac{\gamma\vartheta_k}{n_k} \prod_{i=g+1}^{k} \frac{n_i}{n_i + \gamma\vartheta_i} + \frac{\gamma + r^{\gamma+1} - (\gamma + 1) r}{(\gamma + 1) r (1 - r^{\gamma})} \prod_{i=g+1}^{K} \frac{n_i}{n_i + \gamma\vartheta_i}.$$

Equation (26) then follows from using

$$1-\sum_{k=l+1}^{K}\frac{\gamma\vartheta_k}{n_k}\left(\prod_{i=l+1}^{k}\frac{n_i}{n_i+\gamma\vartheta_i}\right)=\prod_{i=l+1}^{K}\frac{n_i}{n_i+\gamma\vartheta_i}, \tag{32}$$

which can straightforwardly be shown to hold by induction.

Similarly, (8) reduces to

$$\lambda_U=\frac{(1-r)}{r\left(1-r^{\gamma}\right)}\sum_{k=1}^{g-1}\frac{\gamma\vartheta_k}{n_k}\prod_{i=k+1}^{g}\frac{n_i+\gamma\vartheta_i}{n_i}.$$

Equation (27) then follows from using

$$\sum_{k=1}^{g-1}\frac{\gamma\vartheta_k}{n_k}\prod_{i=k+1}^{g}\frac{n_i+\gamma\vartheta_i}{n_i}=\frac{n_g+\gamma\vartheta_g}{n_g}\left(\prod_{i=1}^{g-1}\frac{n_i+\gamma\vartheta_i}{n_i}-1\right).$$

Equations (28)–(30) are derived analogously. ■

Corollary 1 *For linear inverse demand ($\gamma=1$), the measures of harm are given as*

$$\begin{aligned}\lambda_D &= \frac{Q^*+Q^g}{Q^g}\left(1-\frac{1}{2}\prod_{i=g+1}^{K}\frac{n_i}{n_i+\vartheta_i}\right),\ \lambda_U=\frac{n_g+\vartheta_g^c}{n_g}\left(\prod_{i=1}^{g-1}\frac{n_i+\vartheta_i}{n_i}-1\right)\\ \lambda_g &= \frac{\vartheta_g}{n_g}\frac{Q^*}{Q^g}-1,\ \lambda_{g+1}=\frac{\vartheta_{g+1}}{n_{g+1}+\vartheta_{g+1}}\frac{Q^*+Q^g}{Q^g},\ \textit{and}\ \lambda_C=\frac{1}{2}\frac{Q^*+Q^g}{Q^g}\prod_{i=g+1}^{K}\frac{n_i}{n_i+\vartheta_i}.\end{aligned}$$

Proof. This follows immediately from substituting $\gamma=1$ and $\frac{1-r^2}{r(1-r)}=\frac{1+r}{r}=\frac{Q^*+Q^g}{Q^g}$ and $\frac{1-r}{r(1-r)}=\frac{1}{r}=\frac{Q^*}{Q^g}$ into equations (26)–(30). ■

A.2 Proofs of the Main Results

First we establish that for small γ, λ_D can increase without bound.

Proof of Proposition 1. First note that we have $\lim_{\gamma\to\infty} r=1$ and, using $\lim_{\gamma\to 0}(1+\alpha\gamma)^{\frac{1}{\gamma}}=\exp[\alpha]$, we have $\lim_{\gamma\to 0} r=\exp\left[\frac{\vartheta_g-\vartheta_g^c}{n_g}\right]$. Moreover, the following results are useful

$$\begin{aligned}\lim_{\gamma\to\infty} r^{\gamma+1} &= \lim_{\gamma\to\infty}\left(\frac{1+\gamma\frac{\vartheta_g}{n_g}}{1+\gamma\frac{\vartheta_g^c}{n_g}}\right)\lim_{\gamma\to\infty} r=\frac{\vartheta_g}{\vartheta_g^c},\\ \lim_{\gamma\to 0} r^{\gamma+1} &= \lim_{\gamma\to 0}\left(\frac{1+\gamma\frac{\vartheta_g}{n_g}}{1+\gamma\frac{\vartheta_g^c}{n_g}}\right)\lim_{\gamma\to 0} r=\exp\left[\frac{\vartheta_g-\vartheta_g^c}{n_g}\right].\end{aligned}$$

Now it follows immediately that

$$\lim_{\gamma\to\infty}\lambda_D=\lim_{\gamma\to\infty}\frac{1-r^{\gamma+1}}{r-r^{\gamma+1}}\lim_{\gamma\to\infty}\left(1-\frac{1}{\gamma+1}\prod_{i=g+1}^{K}\frac{n_i}{n_i+\gamma\vartheta_i}\right)=1.$$

In order to evaluate $\lim_{\gamma\to 0}\lambda_D$ first notice that λ_D can be rewritten as

$$\begin{aligned}
\lambda_D &= \frac{1-r^{\gamma+1}}{r(1-r^{\gamma})}\left(1-\frac{1}{\gamma+1}\prod_{i=g+1}^{K}\frac{n_i}{n_i+\gamma\vartheta_i}\right) \\
&= \frac{1-r^{\gamma+1}}{r}\frac{n_g+\gamma\vartheta_g^c}{\gamma(\gamma+1)\left(\vartheta_g^c-\vartheta_g\right)}\left(\gamma+1-\prod_{i=g+1}^{K}\frac{n_i}{n_i+\gamma\vartheta_i}\right) \\
&= \frac{1-r^{\gamma+1}}{r}\frac{n_g+\gamma\vartheta_g^c}{\gamma(\gamma+1)\left(\vartheta_g^c-\vartheta_g\right)}\left(\gamma+\frac{\prod_{i=g+1}^{K}(n_i+\gamma\vartheta_i)-\prod_{i=g+1}^{K}n_i}{\prod_{i=g+1}^{K}(n_i+\gamma\vartheta_i)}\right) \\
&= \frac{1-r^{\gamma+1}}{r}\frac{n_g+\gamma\vartheta_g^c}{\gamma(\gamma+1)\left(\vartheta_g^c-\vartheta_g\right)}\left(\gamma+\frac{\gamma\left[\sum_{i=g+1}^{K}\left(\vartheta_i\prod_{j=g+1,\, j\neq i}^{K}n_j\right)+f(\gamma)\right]}{\prod_{i=g+1}^{K}(n_i+\gamma\vartheta_i)}\right),
\end{aligned}$$

where $f(\gamma)$ is a function with the property that $\lim_{\gamma\to 0}f(\gamma)=0$. Taking the limit now gives

$$\begin{aligned}
\lim_{\gamma\to 0}\lambda_D &= \left(\frac{1-\exp\left[\frac{\vartheta_g-\vartheta_g^c}{n_g}\right]}{\exp\left[\frac{\vartheta_g-\vartheta_g^c}{n_g}\right]}\right)\frac{n_g}{\vartheta_g^c-\vartheta_g}\left(1+\frac{\sum_{i=g+1}^{K}\left(\vartheta_i\prod_{j=g+1}^{K}n_j/n_i\right)}{\prod_{i=g+1}^{K}n_i}\right) \\
&= h\left(\frac{\vartheta_g^c-\vartheta_g}{n_g}\right)\left(1+\sum_{i=g+1}^{K}\frac{\vartheta_i}{n_i}\right),
\end{aligned}$$

with $h(x)=\frac{\exp(x)-1}{x}$. Note that $\lim_{x\to 0}h(x)=1$, $h'(x)>0$ and $h(1)=e-1$. Clearly, the second part of the above expression, $\left(1+\sum_{i=g+1}^{K}\frac{\vartheta_i}{n_i}\right)$, is bounded from above by $K-g+1$. Taken together we find that $\lim_{\gamma\to 0}\lambda_D\leq(1+K-g)(e-1)$, with the upperbound being exact when all downstream layers are monopolized (implying $\vartheta_i=n_i$ for all i), and there is full collusion in the colluding layer and perfect competition otherwise ($\frac{\vartheta_g^c-\vartheta_g}{n_g}=1$). ■

Alternatively, one can consider the relation between total downstream harm and total profit loss (i.e. overcharge plus output effect) of the direct purchasers. That is,

$$\widetilde{\lambda}_D=\frac{d_D}{\xi_{g+1}+\sigma_{g+1}}=\frac{\lambda_D}{\Xi},$$

where $\Xi=\frac{\xi_{g+1}+\sigma_{g+1}}{\xi_{g+1}}$. Obviously, $\Xi\geq 1$. Moreover, if layer $g+1$ is perfectly competitive (that is, $\vartheta_{g+1}\to 0$ and/or $n_{g+1}\to\infty$) there is no output effect for this layer of direct purchasers, $\sigma_{g+1}=0$. This implies $\Xi=1$ and $\widetilde{\lambda}_D=\lambda_D$. Obviously, the downstream damage multiplier $\widetilde{\lambda}_D$ can therefore also take on any value. However, this holds even if the indirect purchaser layer is imperfectly competitive as the following lemma shows.

Lemma 3 *For any $\overline{M}>0$ and any value of $\vartheta_{g+1}/n_{g+1}\leq 1$ there exists a market structure such that $\widetilde{\lambda}_D>\overline{M}$.*

Proof. First, using Lemma 2 we find that

$$\Xi=1+\frac{1-r}{r}\frac{\vartheta_{g+1}}{n_{g+1}+\gamma\vartheta_{g+1}}\frac{n_g+\gamma\vartheta_g^c}{\vartheta_g^c-\vartheta_g}.$$

From the proof of Proposition 1 we know that $\lim_{\gamma\to 0} r = \exp\left[\frac{\vartheta_g-\vartheta_g^c}{n_g}\right]$. Using $h(x) = \frac{\exp(x)-1}{x}$ again we can write

$$\lim_{\gamma\to 0}\widetilde{\lambda}_D = \lim_{\gamma\to 0}\frac{\lambda_D}{\Xi} = \frac{h\left(\frac{\vartheta_g^c-\vartheta_g}{n_g}\right)}{1+\frac{\vartheta_{g+1}}{n_{g+1}}h\left(\frac{\vartheta_g^c-\vartheta_g}{n_g}\right)}\left(1+\sum_{i=g+1}^{K}\frac{\vartheta_i}{n_i}\right).$$

Since $h(x)$ is maximized at $x=1$ and $h(1)=e-1$ we find an upper bound for $\widetilde{\lambda}_D$ by taking $\gamma\to 0$, $\vartheta_g=0$, $\vartheta_g^c=n_g$ and $\vartheta_k=n_k$ for $k=g+1,\ldots,K$. This upper bound is given by

$$\frac{e-1}{e}(1+K-g).$$

Clearly, any level of $\widetilde{\lambda}_D$ can be reached by choosing $K-g$ appropriately. ■

Moreover, it is easily verified that for the case with $\vartheta_{g+1}=n_{g+1}$, and a perfectly competitive benchmark in the colluding layer, $\vartheta_g=0$, we have that Ξ equals 2 for $\gamma=1$, and Ξ goes to e (1) for $\gamma\to 0$ ($\gamma\to\infty$)

Next we show that λ_U can increase without bound.

Proof of Proposition 2. It is sufficient to prove the proposition for the linear case with $\gamma=1$. Then we have

$$\lambda_U = \frac{n_g+\vartheta_g^c}{n_g}\left(\prod_{i=1}^{g-1}\frac{n_i+\vartheta_i}{n_i}-1\right).$$

In the extreme case where all upstream layers are monopolized ($\vartheta_i=n_i$ for $i=1,\ldots g-1$) and there is full collusion in the colluding layer ($\vartheta_g^c=n_g$) we obtain

$$\lambda_U = 2\left(2^{g-1}-1\right),$$

which obviously can reach any finite level as the number of upstream layers $g-1$ increases. ■

Finally, we show the effects of the location of the colluding layer on total welfare and the different measures of harm.

Proof of Proposition 3. The change in welfare is

$$\begin{aligned}\triangle W_g &= \sum_{k=1}^{K}\sigma_k+\sigma_C = (Q^*-Q^g)\left(P^*-\sum_{k=1}^{K}c_k\right)+\sigma_C\\ &= \Phi\left[\left(1-\left(\frac{n_g+\gamma\vartheta_g}{n_g+\gamma\vartheta_g^c}\right)^{\frac{1}{\gamma}}\right)-\frac{1}{\gamma+1}\left(1-\left(\frac{n_g+\gamma\vartheta_g}{n_g+\gamma\vartheta_g^c}\right)^{\frac{\gamma+1}{\gamma}}\right)\prod_{i=1}^{K}\frac{n_i}{n_i+\gamma\vartheta_i}\right],\end{aligned}$$

where $\Phi=\left(\frac{1}{b}\right)^{\frac{1}{\gamma}}\left(\prod_{i=1}^{K}\frac{n_i}{n_i+\gamma\vartheta_i}\right)^{\frac{1}{\gamma}}\left(a-\sum_{j=1}^{K}c_j\right)^{\frac{\gamma+1}{\gamma}}$. The change in welfare is independent of the location of the cartel. The direct purchaser overcharge is given by

$$\xi_{g+1}=\omega_g=\Phi\left(\frac{n_g+\gamma\vartheta_g}{n_g+\gamma\vartheta_g^c}\right)^{\frac{1}{\gamma}}\left(\frac{\gamma\vartheta_g^c-\gamma\vartheta_g}{n_g+\gamma\vartheta_g^c}\right)\prod_{i=1}^{g}\frac{n_i}{n_i+\gamma\vartheta_i},$$

which does decrease in g. ■