# Arms or Legs: Isomorphic Dutch Auctions and Centipede Games

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Centipede games and Dutch auctions provide important instances in which game theory fails empirically. The reasons for these empirical failures are not well understood. Standard centipede games and Dutch auctions differ from each other in terms of their Institutional Format (IF), Dynamic Structure (DS), and Information Environment (IE). This paper introduces new games that are constructed from centipede games and Dutch auctions by interchanging some of their IF, DS, and IE characteristics. The new games are introduced in isomorphic pairs. Experiment treatments with pairs of new isomorphic games provide data that yield insights into the effects on behavior of games' IF, DS, and IE characteristics.

## 1. Introduction

Dutch auctions and centipede games both exhibit systematic deviations of behavior in experiments from predictions of game theory. Theory for the centipede game predicts unraveling to a "take" at the first node in the game tree but failure of this prediction is a robust empirical phenomenon (McKelvey and Palfrey, 1992, 1998; Rapoport, et al., 2003; Zauner, 1999). Theory predicts that the Dutch auction is isomorphic to the first price sealed bid auction but prices in a fast-clock Dutch auction are lower and prices in a slow-clock Dutch auction are higher than prices in comparable first price sealed bid auctions (Cox, Roberson, and Smith, 1982; Cox Smith and Walker, 1983; Lucking-Reilly, 1999; Katok and Kwasnica, forthcoming). The reasons for these empirical failures are not well understood. This paper seeks better understanding of the determinants of behavior in centipede games and Dutch auctions by studying new games constructed by interchanging some of the games' typical characteristics.

Standard centipede games and Dutch auctions differ from each other in terms of their Institutional Format (IF), Dynamic Structure (DS), and Information Environment (IE). The IF of the centipede game is an extensive form game tree whereas the IF of the Dutch auction is a price clock. The DS of a centipede game is alternating opportunity to select an action (take or pass) whereas the DS of the Dutch auction is simultaneous opportunity to select an action (bid or don't bid). The IE of the centipede game is public information on (money) payoffs whereas the IE of the Dutch auction is private information on payoffs (because auctioned item values are private). This paper introduces new games that are constructed by interchanging some of the IF, DS, and IE characteristics of the standard games. The paper reports an experiment with the new games that is intended to provide insight into which characteristics of games are empirically significant determinants of behavior and which of these significant characteristics account for systematic differences between theory and behavior.

We construct modified centipede games that have the same IE as standard Dutch auctions: the independent private values environment in which an agent knows her own payoffs but only the probability distribution of others' payoffs for all decision opportunities. We construct modified Dutch auctions that have the same DS as standard centipede games: alternating (rather than simultaneous) opportunities to bid or not bid. We construct modified centipede games that have the same DS as standard Dutch auctions: simultaneous (rather than alternating) opportunities to take or pass. We construct modified centipede games that have the same IF as standard Dutch auctions: a decreasing price clock. Finally, we construct Dutch auctions with the IF that characterizes standard centipede games: an extensive form game tree. The paired clock and tree formats for the modified Dutch auctions are theoretically isomorphic. A theoretical isomorphism also holds for the paired clock and tree formats of the modified centipede games. Because the paired games are strategically equivalent, any significant differences in behavior between them are caused by behavioral significance of characteristics of games that are not captured by existing theory of equilibrium strategies.

## 2. Related Literature

There is a substantial literature on experimental tests of the theory of Dutch auctions and centipede games. We review a few papers.

*2.1 Dutch Auctions*

In his book on the history of auctions, Cassady (1967) discusses auction market formats that have been used for long historical periods. He defines "Dutch auction" as follows (Cassady, 1967, p. 67):

> *In this auction the offer price starts at an amount believed to be higher than any bidder is willing to pay and is lowered by an auctioneer or a clock device until one of the bidders accepts the last offer. The first and only bid is the sales price in the Dutch auction.*

One implication of this institutional format is that any bidder who stops the auction near the beginning is likely to lose money. This is the defining characteristic of "Dutch auction" that we do *not* change when new modified versions are introduced by changing the standard game's IF and DS characteristics.

Vickrey (1961) first explained that the Dutch auction is theoretically isomorphic to the first price sealed bid auction in the independent private values information environment. This implies that the Nash equilibrium bid function is the same function for these two auctions even though the Dutch auction has a decreasing price clock, real time institutional format whereas the first price sealed bid auction has a normal form (time free) game format. Cox, Roberson, and Smith (1982) tested this isomorphism using a Dutch auction price clock with two-second price decrement speed.[1] They reported that the isomorphism failed empirically; the Dutch auction produced lower prices than the first price auction in laboratory experiments using (induced) independent private values. Cox, Roberson, and Smith offered two "real time" models of the Dutch auction, using non-standard assumptions, both of which were consistent with lower prices in the Dutch auction than in the first price auction. One model incorporated a utility from the activity of playing the game that was additive to the utility of money payoff from bidding

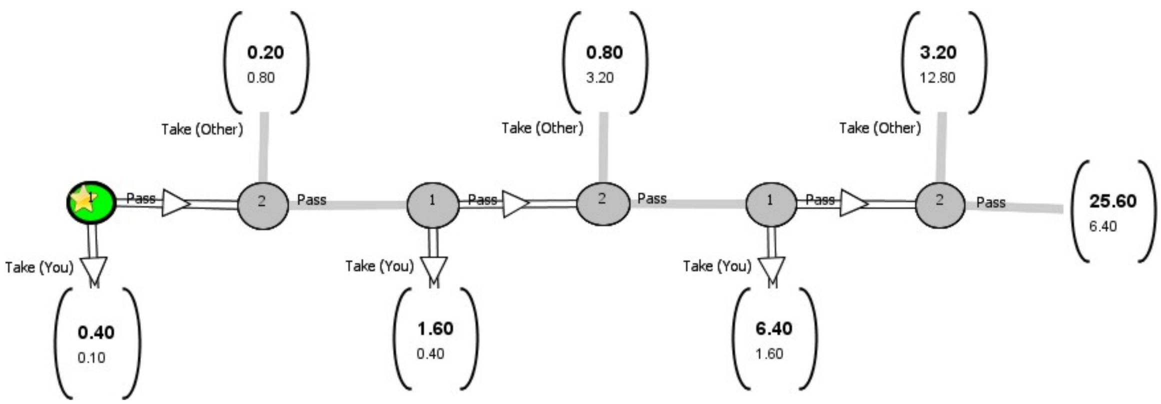
[1] Dutch flower auctions use very fast clock speeds. Characteristics of the biggest flower auction institution are reported at http://www.aalsmeer.nl/00004.asp.

whereas the other model incorporated biased application of Bayes' rule in updating bidders' expectations about rivals' bidding behavior during an auction. Cox, Smith, and Walker (1983) reported an experimental test of the alternative models which led to rejection of the utility of playing the game model in favor of the biased Bayes' rule model.

Lucking-Reiley (1999) reported an internet experiment, with an (natural) uncontrolled information environment, and an extremely slow Dutch price clock. He found that the Dutch and first price auction isomorphism failed but that Dutch prices were higher than prices in the first price auction. Katok and Kwasnica (forthcoming) report an experiment in the independent private values information environment in which Dutch auction clock speed is a treatment variable. They report that the isomorphism fails and that Dutch auction prices are lower (respectively, higher) than prices in the first price auction with a fast (respectively, slow) Dutch price clock. The clock format Dutch auctions and centipede games reported below use a 10-second price clock speed, which is in the middle of the clock speeds used as treatment variables by Katok and Kwasnica (the other two speeds they used were one-second and 30-seconds). The tree format centipede games and Dutch auctions use a comparable 10-second decision opportunity at each node in the extensive form game tree.

### *2.2 Centipede Games*

In a standard centipede game, the payoff of each player is positive at his first decision node. This is the defining characteristic of "centipede game" that we do *not* change when new modified versions are introduced by changing the standard game's IF and IE characteristics. The centipede game is typically implemented in a public information environment in which the payoffs of all players at all decision nodes are public information.  A sequential move structure is standard. Each player has his own decision nodes to choose "take" or "pass." If a player chooses the action "pass," this leads to an increase in the sum of payoffs across players but hands the next "take" opportunity to the other player. A typical centipede game is that used in McKelvey, and Palfrey (1992):




Why study centipede games? The centipede game presents a tension between what economists might suppose to be an agent's wish to obtain a higher payoff for himself, by waiting to take, and a desire to avoid getting zero (or a low) payoff by waiting so long that the other player takes first. This tension within an agent, and the potential for each agent to develop beliefs about this tension (and other possible motivations) in other agents makes the centipede game a potential test-bed for hypotheses on a range of subjects.

For instance, does the predicted subgame perfect outcome (taking at the first node) occur in the laboratory? McKelvey and Palfrey (1992) found that generally it did not. What are the implications of such observations? Can failure to play according to the theoretical prediction be taken as evidence of altruism, decision errors, or beliefs that other altruistic or error-prone agents might be present in the population from which opponents are drawn? A number of researchers have attempted to explain the disparity between the empirical results and the theoretical prediction using one or another such explanation. McKelvey and Palfrey (1992) examine the possibility of the presence of altruistic agents (who prefer their opponents to make more money rather than less) as explaining the empirical failure of the unraveling prediction. McKelvey and

Palfrey (1998) examines the explanatory possibilities of a particular error-in-choice model: quantal response equilibrium (QRE). Zauner (1999) examines the possibility that “independent perturbed payoffs” (IPP) can explain the data (and estimates the magnitude of noise in perceived payoffs needed to do so).

Fitting new models to data from a particular game is one way to attempt to understand what is going on in that kind of game. Another approach is to vary characteristics of the game in order to assess their significance as determinants of behavior. Rapoport, Stein, Parco, and Nicholas (2003) vary parameters of the design of the centipede game (such as the number of players and the magnitude of the payoffs) in order to assess whether the traditional result of failure to unravel is still observed; they find that moving from two to three players, and increasing the size of payoffs appear to increase the incidence of unraveling.

Our paper contributes to the literature by experimenting with the effects of changes in institutional format, dynamic structure, and information environment in new games derived from standard centipede games and Dutch auctions. If changes in IF, DS, or IE produce significant changes in behavior, this would suggest that parameters from models such as QRE or IPP would have to vary across different representations of the same game, thus leading us to reflect upon the generality of those models. This is also why we look at both Dutch auctions and centipede games in the same study, as it allows us to see if any dependence of bidding/takes on game form generalizes across games. Additionally, it should be noted that complete unraveling of the centipede game (however parameterized) with only two players (rather than three as in Rapoport, et al.) and few repetitions would be a new type of empirical result.

### *2.3 Maintained Difference between Dutch Auctions and Centipede Games*

Our experiment implements modified forms of Dutch auctions and centipede games that are constructed by changing some of the IF, DS, and IE characteristics of the traditional games. This suggests the question of what characteristic(s) of the modified games separate centipede games from Dutch auctions. We maintain one distinction between the two types of games: the games are always parameterized so that theory predicts unraveling to the first decision opportunity (tree node or clock tick) in any centipede game whereas theory predicts interior (to the decision space) Nash equilibrium in any Dutch auction.

### *2.4 Varying Game Forms*

What accounts for the failure of Nash equilibrium bidding theory to predict prices in Dutch auctions? Previous literature has offered two explanations: (a) underestimation of the risk of letting the clock continue to run as, for example, in the Bayes' rule miscalculation model (Cox, Roberson, and Smith, 1982; Cox, Smith, and Walker, 1983); and (b) bidder impatience as, for example, in the model in Katok and Kwasnica (forthcoming). The sequential bid form of the Dutch auction that we implement may make the risk from letting the clock continue to run be more salient. Furthermore the sequential bid form of the Dutch auction may make impatience more pronounced by doubling (from one to two) the number of clock ticks between successive bid opportunities. Our experimental design does not address the distinction between the possible effects of Bayes rule violation and impatience. Instead, we ask whether failure of Nash equilibrium bidding theory is robust to changes in the IF and DS of the bidding game.

What accounts for the empirical failure of the unraveling prediction in standard centipede games? Is it the public information environment, which makes salient the exact opportunity cost of the other player from passing at each node? We introduce the independent private values form of the centipede game, which makes it impossible for a player to know the other player's exact opportunity cost at any decision node but preserves the pattern of increasing opportunity costs for both players.

**3. Arms and Legs: Clock and Tree Formats with Sequential or Simultaneous Moves**

We explain clock and tree formats of Dutch auctions and centipede games with independent private values. Each (clock or tree) format of the Dutch auction or centipede game is developed with alternative dynamic structures involving sequential or simultaneous move (bid or take) opportunities.

*3.1. Sequential Move Dutch and Centipede Clock Games*

The sequential-bid Dutch auction with clock format is simply the traditional Dutch auction with the one change that the bidders alternate price clock ticks at which they are allowed to bid. That is, for a given clock reading, only one bidder has the right to bid at that clock price; the other bidder(s) would have to wait until the clock counts down to their turn before they would have a chance to bid. In the two bidder, $1.00 price clock decrement case we utilize in our design, one bidder can bid at price clock readings of 10,8,6,4, or 2, while the other bidder can buy at clock readings of 9,7,5,3, or 1. The bidders' private values for the auctioned item are independently drawn from the uniform distribution on [1.01, 1.02, …, 10.99, 11.00]. This version of the Dutch auction is presented to bidders as follows.

-----------------------INSERT SEQUENTIAL DUTCH SCREEN SHOT HERE-----

**Screen shot clock form (Duncan and Kevin)**

Include screen shots in a sequence demonstrating passage of time

The sequential-take IPV centipede game with clock format maintains the DS of alternating decision opportunities that characterizes the traditional game but incorporates that DS into a price clock IF. The IPV centipede game in clock format (and tree format) replaces the public information IE of the traditional centipede game with private information about players' payoffs. In the two-player, $1.00 price tick decrement game we utilize in our design, one player can take at price clock readings of 10, 8, 6, 4, or 2 while the other player can take at price clock readings of 9, 7, 5, 3, or 1. The players' private values are independently drawn from the uniform distribution on [11.01, 11.02,…, 20.99, 21.00]. This version of the centipede game is presented to subjects as follows.

INSERT CLOCK FORMAT CENTIPEDE GAME SCREEN SHOT HERE

### *3.2 Sequential Move Centipede and Dutch Tree Games*

We utilize an independent private values information environment for our version of the sequential-take centipede game in tree format (as well as clock format). Each player has an initial value drawn from a uniform distribution on [0.01, 0.02, …, 9.99, 10.00], to which an amount *n* is added at each subsequent decision node $n$ = 1, 2,…, 10. A player's payoff from taking at node *n* (when possible) is $v_j + n$, where $v_j$ is the player's private value. Each player can only see that the other player has some unknown initial value "*v* not *i*," to which is added \$2.00 at each of the subsequent *other player's* decision nodes. The centipede game in tree format is presented to the subjects as follows.

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The sequential-bid Dutch auction in tree format (as well as clock format) uses the DS of alternating bid opportunities instead of the simultaneous bid opportunities of the traditional Dutch auction. The tree format Dutch auction represents the auction with an extensive form game tree rather than a price clock. Each bidder has an initial value drawn from the uniform distribution on [-9.99, -9.98, …, -0.01, 0.00], to which an amount *n* is added at each subsequent decision node $n$. A bidder's payoff from bidding at a node $n$ (when possible) is $v_j + n$, where $v_j$ is the bidder's private value for the auctioned item. As in traditional IPV Dutch auctions (Vickrey, 1961), each bidder knows his own auctioned item value but only knows the distribution from which the other bidder's value is drawn. The Dutch auction in tree format is presented to the subjects as follows.

INSERT DUTCH AUCTION TREE FORM SCREEN SHOT HERE

### *3.3 Simultaneous Move Dutch and Centipede Clock Games*

The simultaneous-bid Dutch auction in clock format is the standard form of that auction. The simultaneous-take IPV centipede game in clock format differs from the standard centipede game in IF, DS and IE: the clock format is used; each player can take at every node; and players' payoffs are private information.

### *3.4 Simultaneous Move Centipede and Dutch Tree Games*

The simultaneous-bid Dutch auction in tree format differs from the standard Dutch auction only in its IF: a tree format is used rather than a clock format. The simultaneous-take IPV centipede game with tree format differs from the standard centipede game in DS and IE: each player can take at every node and players' payoffs are private information.

## **4. Theoretical Predictions**

For the two games in each pair to be isomorphic, they need to have the same dynamic structure. Implementing this structure entailed a list of features. First, we made the role of time identical across clock format and tree format; that is, both formats allowed subjects 10 seconds at a given price clock reading or decision node in the tree in which the subject could click with a mouse on a bid or take.[2] If at the end of 10 seconds the subject had not bid or taken, the game advanced to the next price clock reading or decision node in the tree; the subject could not actively select "not to bid" or "not to take" but could only do so by waiting 10 seconds for the computer to advance the game. Second, we set the change in payoff from one decision opportunity to the next one (belonging to the other player) equal to $1.00 across all games in all sessions. Third, all games

[2] Data reported in Katok and Kwasnica (forthcoming) make clear that clock speed affects outcomes in Dutch clock auctions. This suggests that clock speed could affect outcomes in centipede clock games and that node decision time could affect outcomes in Dutch and centipede tree games.

were constructed so as to have a default setting of zero payoffs for both players in the event of "no bid" in the auction or "no take" in the centipede game. We did this because we needed the same end-of-game payoffs across games to preserve isomorphism and because the default of "no transaction" is what is natural for Dutch auctions. Also, the fact that some existing centipede experiments were structured with a (0,0) terminal node encouraged us in settling on this standardization.

*4.1 Predictions for the Sequential-Bid Dutch Auction with Clock Format*

In our sequential Dutch auction with clock format, the odd node bidder can bid at clock prices of 9, 7, 5, 3, and 1. The even node bidder can bid at clock prices of 10, 8, 6, 4, and 2. The two bidders' values for the auctioned item are independently drawn from the uniform distribution on [1.01, 1.02, …, 10.99, 11.00]. The item values and bids are (of necessity) discrete, hence the bid functions are step functions. The risk neutral Nash equilibrium bid functions for odd-node and even-node bidders are as follows. Let $b_s^o[l_s^o, h_s^o]$ denote that values in the range $[l_s^o, h_s^o]$ support a Nash equilibrium bid at price $b_s^o$ on the Dutch price clock by the odd-node bidder, for whom $b_s^o \in \{9,7,5,3,1\}$. Similarly, let $b_t^e[l_t^e, h_t^e]$ denote that values in the range $[l_t^e, h_t^e]$ support a Nash equilibrium bid at price $b_t^e$ on the Dutch price clock by the even-node bidder for whom $b_t^e \in \{10,8,6,4,2\}$. No values support bids of 9 or 7 by the odd-node bidder. Similarly, no values support bids of 10 or 8 by the even-node bidder. The other parts of the of the Nash equilibrium bid functions are:

Odd Node Bidder: 5[7.25,11.00], 3[3.59,7.24], 1[1.01,3.58]

Even Node Bidder: 6[9.32,11.00], 4[5.41,9.31], 2[2.00,5.40]

The bid functions were solved for numerically. The numerical procedure consists of:

(1) Initializing each bidder's probability of bidding at each of the five clock prices at which they can bid at 1/5.

(2) Calculating the expected value maximizing bids for each bidder, given the probabilities of winning (at each price) implied by the opposing bidder's probabilities of bidding in step (1).

(3) Updating the probability of bidding at a given price (for each bidder) based on the expected value maximizing bids in step (2).

(4) Looping through steps (2) and (3) until mutual best response bid functions are obtained for both bidders.[3]

*4.2 Predictions for the Sequential-Bid Dutch Auction with Tree Format*

The same odd bidder and even bidder Nash equilibrium bid functions apply to the tree format of the sequential Dutch auction, as can be seen from the following. Let $X_j, j = 1,2,$ denote the random variables for the two bidders' item values in the clock format auction. Let $x_j$, $j = 1,2,$ denote the two bidder's realized values in the auction. Bidder $j$ knows that her own payoff from bidding at price clock tick $t$ when it is her turn is $x_t - 11 + t$ (because the clock price is 10 at tick 1). Bidder $j$ does not know bidder $k's$ ($k \neq j$) payoff from bidding at any permissible tick $\tau$ ; instead, it is the random variable $X_k - 11 + \tau$.

Let $Y_j$, $j = 1,2,$ denote the random variables for the two bidders' item values in the tree format Dutch auction and let $y_j$, $j = 1,2,$ denote their realized values. Bidder $j$ knows that his own value from bidding at any node $n$ (when it is his turn) is $y_j + n$. Bidder $j$ does not know bidder $k's$ ($k \neq j$) value from bidding at node $\eta$; instead, it is the random variable $Y_k + \eta$.

[3] It should be noted that this procedure involves some potential for rounding error. However, because realized items values are rarely at boundaries of the bid function segments, the conclusions from data analysis would not be affected.

The clock format and tree format Dutch auctions are isomorphic, and hence have the same equilibrium bid functions, because of their different uniform distributions of values. At tick $t = s$ and node $n = s$, the random payoffs from bidding in the clock and tree format auctions are, respectively, $X_j - 11 + s$ and $Y_j + s$. Recall that $X_j$ is uniformly distributed between 1.01 and 11.00 whereas $Y_j$ is uniformly distributed between -9.99 and 0.00. Therefore, at any tick = node = $s$, the probability distributions of $X_j - 11 + s$ and $Y_j + s$ are identical.

*4.3 Predictions for the Sequential-Take IPV Centipede Game with Tree Format*

The tree format centipede game with sequential take opportunities and independent private values has a subgame perfect equilibrium similar to that associated with the traditional public value centipede game. This can be seen as follows. Let the two players have any two values, $v_1$ and $v_2$ drawn from the support [0.01, 0.02, …, 9.99, 10.00] for the uniform distribution of values. This creates a situation where the first mover could earn \$$v_1$+1 at her first take node, \$$v_1$+3 at her next take node, then \$$v_1$+5, \$$v_1$+7, or \$$v_1$+9 at her successive take nodes, while the second mover could earn \$$v_2$+2, \$$v_2$+4, \$$v_2$+6, \$$v_2$+8, or \$$v_2$+10 at her (respective) take nodes. The player who does not take earns zero at all nodes. If neither player has taken by the time the second mover's final take node has timed out (after 10 seconds), both players earn zero. This suggests that if the second mover's final take node were to be reached, the second mover would take and would earn \$$v_2$+10 – while the first mover would receive zero. A rational first mover would anticipate this, and take at the preceding node; but a rational second mover should in turn anticipate this, and take at the preceding node, and so on. This argument leads to a take at the first node by the first mover. Therefore, the theoretical prediction for the IPV centipede game is unraveling to a take at the first node, the same as theory predicts for the traditional centipede game with public values.

*4.4 Predictions for the Sequential-Take IPV Centipede Game with Clock Format*

The same (unraveling) subgame perfect equilibrium applies to the clock format centipede game, as shown by the following. Let $Y_j$, $j=1,2,$ denote the two players' random values in the tree format centipede game and let $X_j$, $j=1,2,$ denote their values in the clock format game. Let $y_j$ and $x_j$, $j=1,2,$ denote the players' realized values. Player $j$ knows that his own payoff from taking at node $n$ in the tree game (when it is his turn) is $y_j+n$. Player $j$ does not know player $k's$ ($k \neq j$) payoff from taking (when permitted) at node $\eta$ in the game tree; instead, it is the random variable $Y_k+\eta$. Similarly, player $j's$ payoff from taking at tick $t$ in the clock format centipede game is $x_j-11+t$ (because the price on the clock is 10 at tick 1). Player $j$ only knows that player $k's$ ($k \neq j$) payoff from taking at tick $\tau$ in the clock format game is the random variable $X_k-11+\tau$.

The tree format and clock format IPV centipede games have the same (unraveling) equilibrium because of their different uniform distributions of values. At node $n=s$ and tick $t=s$, the random payoffs from taking in the tree format and clock format games are, respectively, $Y_j+s$ and $X_j-11+s$. Recall that $Y_j$ is uniformly distributed between 1.01 and 11.00 whereas $X_j$ is uniformly distributed between 11.01 and 21.00. Therefore, at any node = tick = $s$, the probability distributions of $Y_j+s$ and $X_j-11+s$ are identical.

*4.5 Predictions for the Simultaneous-Bid Dutch Auction with Clock Format*

The risk neutral bid functions for the simultaneous Dutch with clock format and values drawn from a uniform distribution on [1.01, 1.02, …, 10.99, 11.00] and with each player able to bid at 10,9,8,7,6,5,4,3,2, or 1 are as follows. Let $b_s[l_s, h_s]$ denote that values in the range $[l_s, h_s]$

support a Nash equilibrium bid at price $b_s$ on the Dutch price clock. No values support bids of 10, 9, 8, 7, or 6. The other parts of the of the Nash equilibrium bid function are:

Either Bidder: 5[7.67,11.00], 4[6.46,7.66], 3[4.16,6.45], 2[2.47,4.15], 1[1.01,2.46]

The bid functions were solved for numerically. This was done by means of applying the FindRoot routine in Mathematica 6.0 to a system of equations embodying Conditions 1 through 4 in Chwe (1988). Chwe's Conditions 1 through 4 together provide for characterizing the pure strategy Nash equilibrium bid function in a first price sealed bid auction with a discrete bid space. The numerical solution satisfies the inequalities in Chwe's Proposition 1. The isomorphism between the first price sealed bid auction and the Dutch auction is then invoked to apply that solution in the present setting.

### *4.6 Predictions for the Simultaneous-Bid Dutch Auction with Tree Format*

Arguments similar to those in subsections 4.2 and 4.5 can be used to derive the Nash equilibrium bid function for the simultaneous-bid Dutch auction with tree format. With initial values $X_j$, $j$ = 1,2 drawn from the uniform distribution on [-9.99, -9.98, …, -0.01, 0.00] and payoffs for exiting at node $n$ given by $X_j + n$, the Nash equilibrium strategy function is as follows. Let $t_n[l_n, h_n]$ denote that values in the range $[l_n, h_n]$ support a Nash equilibrium take at node $n$. No initial values correspond to taking at any of the first five nodes. The other parts of the Nash equilibrium strategy function are:

Either Player: 6[-3.33,0.00], 7[-4.54,-3.34], 8[-6.84,-4.55], 9[-8.53,-6.85], 10[-9.99,-8.54]

### *4.7 Predictions for the Simultaneous-Take IPV Centipede Game with Tree Format*

The tree format centipede game with simultaneous take opportunities and independent private values has a subgame perfect equilibrium similar to that associated with the traditional *public value* centipede game with *sequential* take opportunities. This can be seen as follows. Let the two players have any two values, $v_1$ and $v_2$ drawn from the support [0.01, 0.02, …, 9.99, 10.00] for the uniform distribution of values. This creates a situation where player *a* would earn $v_a + t$ if she successfully takes at node $t$ = 1, 2, …, 10 while player *b* would earn $v_b + t$ if he successfully takes at node *t*. The player who does not successfully take earns zero. If both players try to take at the same node, the probability that either succeeds in taking (by entering the take response first) is 1/2. If neither player has taken by the time the final take node has timed out (after 10 seconds), both players earn zero. Therefore if node 10 (the final take node) were to be reached, both players would want to take at that node, and the probability that either would succeed would by 1/2. A rational player would anticipate this, and consider the expected payoff from taking at node 9. If player *k* does *not* try to take at round 9 then player *j* prefers to take because she would succeed with probability 1 and receives payoff $v_j + 9$, which is greater than the expected payoff from waiting until node 10, which is 1/2 $\times(v_j + 10)$. If player *k* does try to take at round 9 then player *j* prefers to take because she would succeed with probability 1/2 and receive expected payoff 1/2 $\times(v_j + 9)$, which is greater than the zero amount that would be received from not trying to take at round 9. Therefore each player prefers to take rather than pass at round 9. Similar reasoning shows that each player prefers to take rather than pass at round 8, and so on back to round 1. Therefore, the theoretical prediction for the simultaneous move IPV centipede game is unraveling to a take at the first node, the same as theory predicts for the traditional centipede game with sequential moves and public values.

### *4.8 Predictions for the Simultaneous-Take IPV Centipede Game with Clock Format*

Arguments similar to those in subsections 4.3 and 4.7 can be used to derive the subgame perfect equilibrium strategy function for the simultaneous-take centipede game with clock format. With initial values $x_j$, $j$ = 1,2 drawn from the uniform distribution on [11.01, 11.02, …, 20.99, 21.00] and payoffs for taking at clock tick $t$, with price equal to $p_t = 11 - t$, for $t = 1, 2, \cdots, 10$, equal to $x_j - p_t$ unravels to a take at the first tick. This then means each player always has a (strictly) dominant strategy involving taking at a price equal to 10 (the first tick at which players can take).

**5. Experimental Design**

In this section we describe details of the experimental design. In section 3 we described how to implement two (isomorphic) versions of each game. We now explain how non-essential differences across different games are eliminated by judicious parameterization. We present a matrix of treatments and a discussion of treatment sequencing to show how all of this allows for an assessment of how observed behavior can be inconsistent with theoretical isomorphisms and how such inconsistencies, if observed, can be attributed to behavioral properties of alternative institutional formats.

In addition to design choices intended primarily to hold constant dynamic structure across institutional formats, we also made certain design choices in order to minimize the differences in parameterization across treatments. We did this so as to minimize potential sources of confounds. First, we used an 11 tick clock or 11 node tree in every treatment. We did this initially in order to allow for a wider range of behavior, particularly in the sequential Dutch auction since, as the number of price clock ticks goes to zero, detection of deviation from the risk neutral Nash equilibrium prediction becomes impossible. Having done this for the sequential Dutch auction, it was necessary to implement a comparable feasible set of choices for the centipede game, so as to better allow comparisons across games. Second, all games utilized an independent private values information environment; again, this removes a potential confounding difference for interpreting results across games. Third, all games utilized a uniform distribution with a support width of \$10.00 as the source of independent private values; again, this standardization removes an impediment to comparisons across games. Finally, it should be pointed out that when all of these design choices were implemented, it left a design where all it took to switch between the sequential Dutch auction and the IPV centipede game was a $10.00 shift in the location (low value, high value, or midrange) of the $10.00 wide uniform distribution used to generate independent private values.

The experimental design was from the outset arranged so as to allow for two key objectives to be fulfilled. First, the design was intended to allow for the detection of violation of theoretical isomorphisms – between the clock and tree formats of the sequential Dutch auction, or between the clock and tree formats of the IPV centipede game. Second, the design allowed for documentation of whether any such violation can be attributed to institutional format; in particular, the design allowed us to ascertain whether any difference between clock and tree formats was similar in Dutch auctions and centipede games.

The experiment included the treatments in the following table.

| | **Sequential IPV Centipede Games** | **Sequential Dutch Auctions** | **Simultaneous IPV Centipede Games** | **Simultaneous Dutch Auctions** |
|---|---|---|---|---|
| Tree Format | Treatment 1 | Treatment 3 | Treatment 5 | Treatment 7 |
| Clock Format | Treatment 2 | Treatment 4 | Treatment 6 | Treatment 8 |

If there is a difference in the distributions of bids for T3 and T4 the isomorphism across institutional formats of the sequential Dutch auction fails. If there is a difference in the distributions of takes between T1 and T2, the isomorphism across institutional formats of the centipede game with independent private values fails. If such failure is observed, are there any generalizable regularities associated with that failure? For instance, if different distributions of bids are observed for T3 and T4, which distribution is closer to the Nash equilibrium prediction? If T1 and T2 yield different distributions of takes, which is associated with earlier takes? And does either the clock format or the tree format lead to earlier or later bids or takes across games, such that this aspect of the results would appear to be driven by institutional format rather than characteristics of the games captured by existing theory?

With this in mind, the details of sequencing of our design were as follows. Each experiment session took approximately 2 hours. In each session, we ran subjects through three treatments with 10 rounds in each treatment. The first, second, and third sets of 10 rounds are referred to, respectively, as Parts 1, 2, and 3. We used the following sequences of treatments in Parts 1, 2, and 3: T3-T4-T3, T3-T2-T3, T4-T3-T4, T4-T1-T4, T2-T1-T2, T2-T3-T2, T1-T2-T1, and T1-T4-T1. For example, the first listed treatment sequence consisted of 10 rounds of T3 in Part 1, followed by 10 rounds of T4 in Part 2, followed by 10 rounds of T3 in Part 3. We experimented with these treatment sequences so as to allow for detection of as full as possible set of potential sequencing effects, and to provide for a variety of across-subjects and within-subjects comparisons to be made. We expressly did not run T1-T3-T1, T3-T1-T3, T2-T4-T2, or T4-T2-T4 sequences, however. This was because for these latter sequences, the only difference in instructions between treatments is the value support – all other text in the instructions remains the same – and we were concerned that subjects might mistakenly think the instructions were exactly the same, potentially even neglecting to read them.

Finally, two brief miscellaneous points regarding the design. We used an integer clock price decrement and node payment increment in all treatments to facilitate quick recognition of payoff information by subjects. We used a 2 to 1 experimental dollar to U.S. dollar exchange rate; this was needed to keep payments affordable if we were going to keep using a $10.00-wide uniform distribution for value draws. (We did not just shrink all numbers by 50% to keep payoffs affordable, as we wanted integer payoff changes between decision opportunities, as noted above.)

## 6. Results

An experiment was run in the laboratory of the Experimental Economics Center (E*x*CEN) at Georgia State University during April 2007. The data are as follows.

*6.1 Sequential Dutch Results*

The data show a dramatic difference in behavior across isomorphic versions of the sequential Dutch auction employing different institutional formats. This can be seen in a number of ways. First of all, one can pool data across all sessions within a given part for the same treatment (e.g. all Part 1 rounds for T4 from all experiment sessions are analyzed as belonging to one distribution) and perform rank-sum tests to ascertain whether we can reject the hypothesis that the distribution of prices for T4 is the same as that for T3. The rank-sum tests performed on pooled data suggest that the distribution of bids is indeed different across the different institutional formats in T3 and T4.[4]

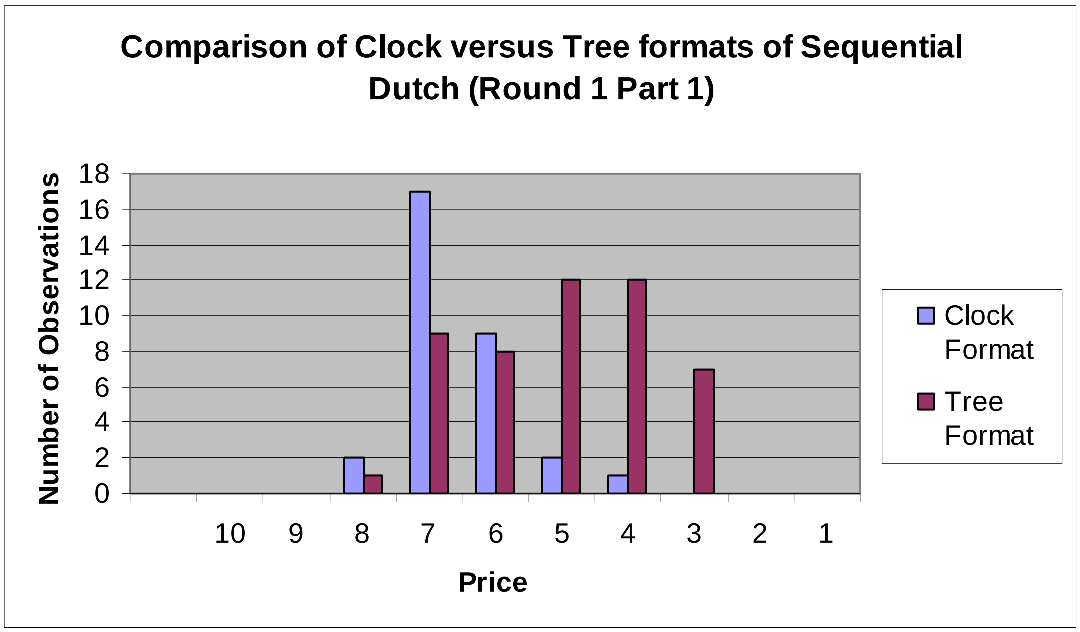
**Rank-Sum Test for Difference in Distributions between T3 and T4 (p-values)**

| Round | Part 1 | Part 2 | Part 3 |
|---|---|---|---|
| 1 | 0.000 | 0.114 | 0.028 |
| 2 | 0.002 | 0.062 | 0.250 |
| 3 | 0.000 | 0.035 | 0.001 |
| 4 | 0.000 | 0.411 | 0.027 |
| 5 | 0.605 | 0.247 | 0.000 |
| 6 | 0.000 | 0.087 | 0.005 |
| 7 | 0.219 | 0.183 | 0.004 |
| 8 | 0.007 | 0.093 | 0.004 |

[4] These tests potentially understate the difference between T4 and T3. There are particular rounds where it is practically impossible to detect any difference in behavior across treatments because the drawn values are extremely low for all of the subjects. For example, if the Even bidder has a value of, say 2.47, and the Odd bidder has a value of 1.80, then almost all the data is going to be in the form of a spike at $2.00 (barring errors) in both treatments. In such a round, it would be impossible to detect a difference in subjects' approach to bidding (across institutional game formats) even if it otherwise existed. Round 5 in Part 1 is one such round.

| | | | |
|---|---|---|---|
| 9 | 0.408 | 0.072 | 0.030 |
| 10 | 0.005 | 0.011 | 0.000 |

To further illustrate the nature of the results, consider the following plot of results from round 1 of part 1 (for which the table above gives a p-value of 0.000 for the rank-sum test of difference in distribution). Every time the reader sees a p-value less than 0.05 in the table of results, the plot of the data underlying that result looks roughly like this:




Clearly, the distributions of bids are very different across institutional formats.

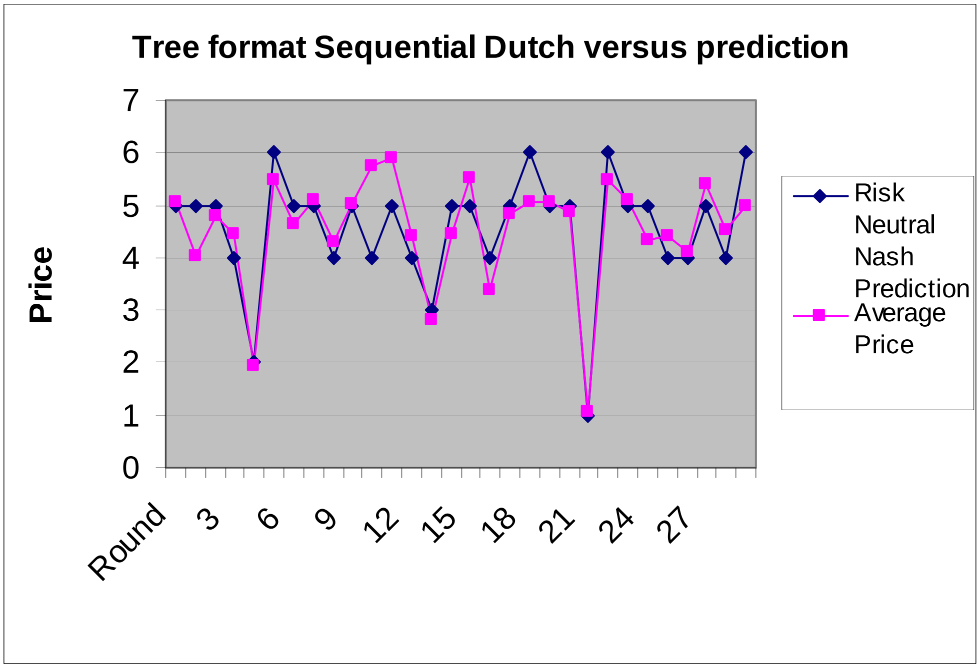
In addition to comparing behavior across institutional formats, we can compare behavior from a single format to the risk neutral Nash equilibrium predicted bids. For the tree format of the sequential Dutch auction, we find the following.

**Rank-Sum Test for Difference in Distribution between T3 and risk neutral**

**Nash equilibrium predictions (p-values)**

| Round | Part 1 | Part 2 | Part 3 |
|---|---|---|---|
| 1 | 0.841 | 0.000 | 0.494 |
| 2 | 0.000 | 0.000 | 0.155 |
| 3 | 0.104 | 0.007 | 0.001 |
| 4 | 0.001 | 0.022 | 0.494 |
| 5 | 0.242 | 0.004 | 0.000 |
| 6 | 0.001 | 0.007 | 0.039 |
| 7 | 0.022 | 0.000 | 0.800 |
| 8 | 0.829 | 0.461 | 0.020 |
| 9 | 0.076 | 0.000 | 0.001 |
| 10 | 0.466 | 0.443 | 0.000 |

In 13 of 30 rounds, we cannot reject the hypothesis that the distribution of prices is the same as the predictions (at the 5% level); in the other 17 rounds behavior is roughly symmetrically distributed between over-bidding and under-bidding. A time series plot of the data makes this clear:



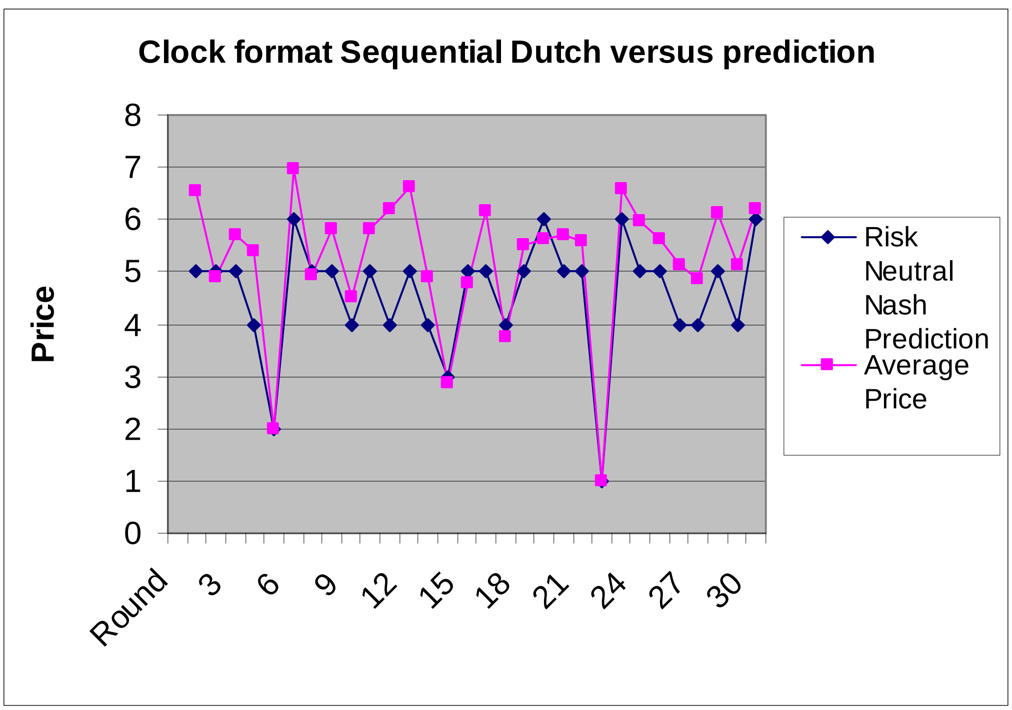

We can perform a similar analysis for the clock format of the sequential Dutch auction, comparing bids therein to the predictions from section 4.1.

**Rank-Sum Test for Differences in Distributions between T4 and risk neutral Nash equilibrium predictions (p-values)**

| Round | Part 1 | Part 2 | Part 3 |
|---|---|---|---|
| 1 | 0.000 | 0.000 | 0.002 |
| 2 | 0.632 | 0.000 | 1.000 |
| 3 | 0.000 | 0.000 | 0.018 |
| 4 | 0.000 | 0.154 | 0.001 |
| 5 | 0.317 | 0.281 | 0.008 |
| 6 | 0.000 | 0.000 | 0.000 |

| | | | |
|---|---|---|---|
| 7 | 0.745 | 0.172 | 0.000 |
| 8 | 0.000 | 0.103 | 0.000 |
| 9 | 0.011 | 0.089 | 0.000 |
| 10 | 0.001 | 0.003 | 0.125 |

In 10 of 30 rounds, we cannot reject (at 5% significance) the hypothesis that the distribution of prices is the same as the predicted distribution; in the other 20 rounds behavior is strongly biased towards over-bidding. A time series plot of the data makes this clear.



*6.2 Centipede Results*

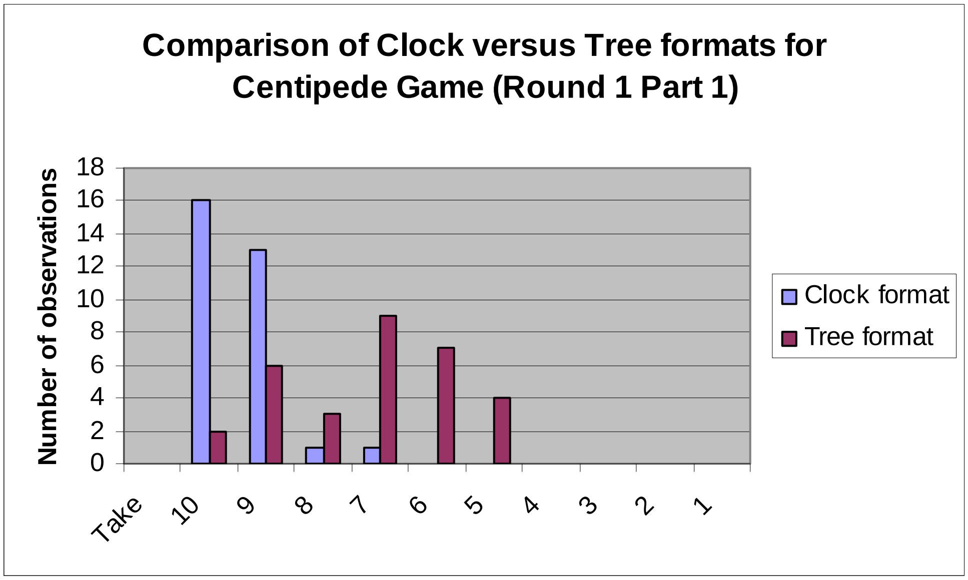
Behavior is also different across institutional formats for the IPV centipede game. Again pooling data across sessions, we find that rank-sum tests generally reject the hypothesis that the distribution of takes is the same across treatments.

**Rank-Sum Test for Differences in Distributions between T1 and T2**

**(p-values)**

| Round | Part 1 | Part 2 | Part 3 |
|---|---|---|---|
| 1 | 0.000 | 0.873 | 0.000 |
| 2 | 0.000 | 0.000 | 0.000 |
| 3 | 0.000 | 0.046 | 0.000 |
| 4 | 0.000 | 0.000 | 0.967 |
| 5 | 0.000 | 0.413 | 0.001 |
| 6 | 0.000 | 0.137 | 0.021 |
| 7 | 0.000 | 0.000 | 0.065 |
| 8 | 0.000 | 0.200 | 0.040 |
| 9 | 0.001 | 0.346 | 1.000 |
| 10 | 0.000 | 0.356 | 1.000 |

In order to better demonstrate the statistical results, consider the following plot comparing results across institutional formats within a round (round 1 of part 1) where the hypothesis that the distributions are the same is rejected with a p-value of 0.000. The results will look similar for other comparisons with p-values less thasn 0.05.




Once again, in addition to comparing data across institutional formats, we can also compare data from a single format to the theoretical prediction (of taking at the first node). For the tree format of the IPV centipede game:

**Rank-Sum Test for Differences in Distributions between T1 and subgame perfect equilibrium (p-values)**

| Round | Part 1 | Part 2 | Part 3 |
|---|---|---|---|
| 1 | 0.000 | 0.000 | 0.000 |
| 2 | 0.000 | 0.000 | 0.000 |
| 3 | 0.000 | 0.021 | 0.000 |
| 4 | 0.000 | 0.000 | 0.078 |
| 5 | 0.000 | 0.154 | 0.000 |
| 6 | 0.000 | 0.078 | 0.021 |
| 7 | 0.000 | 0.000 | 0.000 |

| | | | |
|---|---|---|---|
| 8 | 0.000 | 0.154 | 0.040 |
| 9 | 0.000 | 0.011 | 1.000 |
| 10 | 0.000 | 0.078 | 1.000 |

In 23 out of 30 rounds we reject (at 5% significance) the hypothesis that the distribution of data is the same as the theoretical prediction. Failure to reject the hypothesis occurs more often in later rounds.

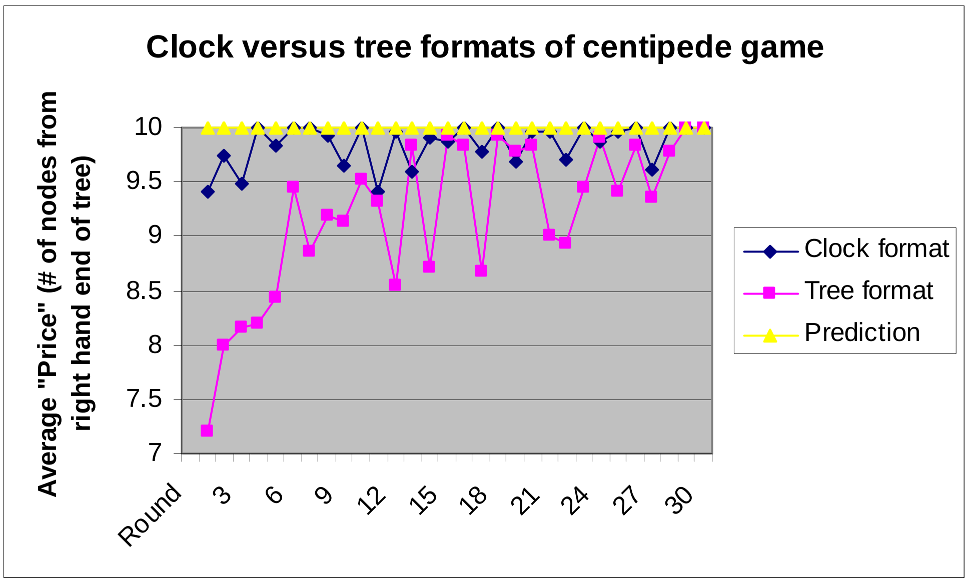
For the clock format of the IPV centipede game, we find very different results. In this format, the equilibrium prediction is attained early and often.

**Rank-Sum Test for Difference in Distributions between T2 and subgame perfect equilibrium (p-values)**

| Round | Part 1 | Part 2 | Part 3 |
|---|---|---|---|
| 1 | 0.000 | 0.000 | 0.317 |
| 2 | 0.003 | 0.317 | 0.001 |
| 3 | 0.000 | 0.001 | 1.000 |
| 4 | 1.000 | 0.153 | 0.078 |
| 5 | 0.021 | 0.076 | 0.317 |
| 6 | 1.000 | 1.000 | 1.000 |
| 7 | 1.000 | 0.153 | 0.000 |
| 8 | 0.154 | 1.000 | 1.000 |
| 9 | 0.000 | 0.004 | 1.000 |
| 10 | 1.000 | 0.317 | 1.000 |

In 10 out of 30 rounds we reject (at 5% significance) the hypothesis that the distribution of data is the same as the theoretical prediction. We observe 5 rejections in Part 1, 3 in Part 2, and only 2 in Part 3.

In order to better illustrate the statistical results pertaining to the centipede game, the following graph plots the average "take" over time, in each format, against the theoretical prediction.



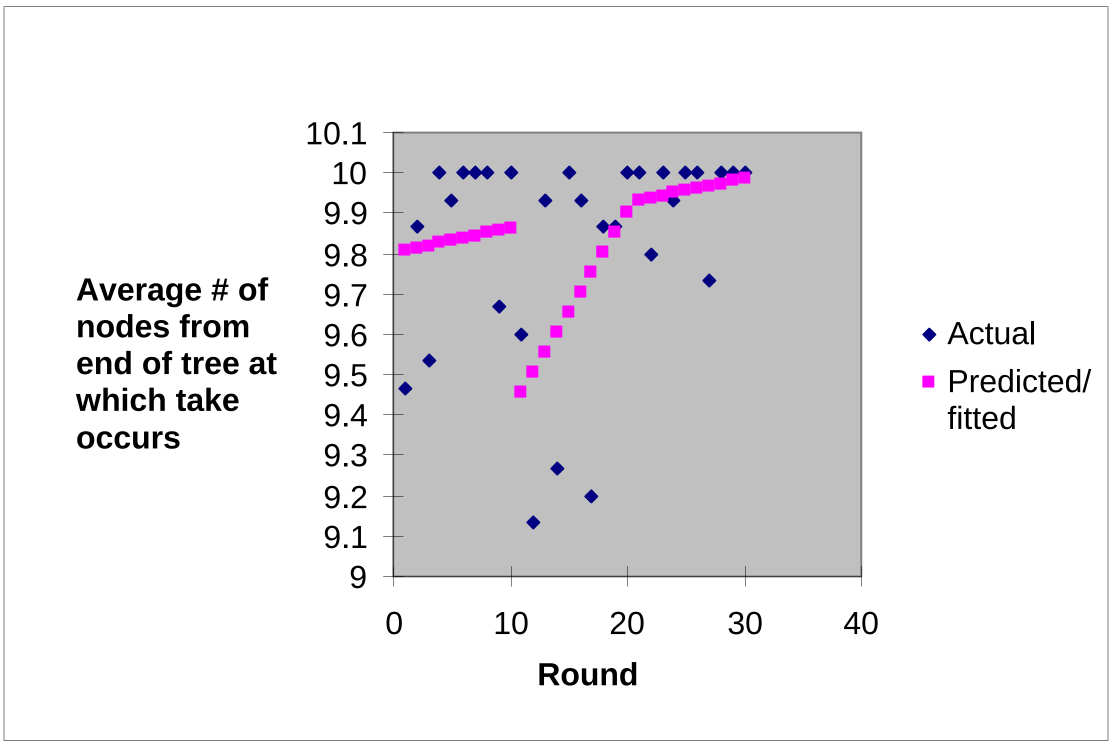

.

In general, we find that the difference in behavior persists until comparatively many repetitions of the game have been completed, at which point T2 starts to reliably achieve the theoretical prediction that T1 attained much earlier. In fact, that T1 was able to attain the theoretical prediction so early is in itself a novel result in the empirical literature on the centipede game.

Finally, in addition to the striking across-subjects results just discussed, the experimental design allows for within-subjects comparisons to be made as well. The T1-T2-T1 and T2-T1-T2 sessions each allow for a regression analysis in which slope and intercept treatment dummies can detect structural shifts in an underlying time time trend (of convergence to the subgame perfect equilibrium).

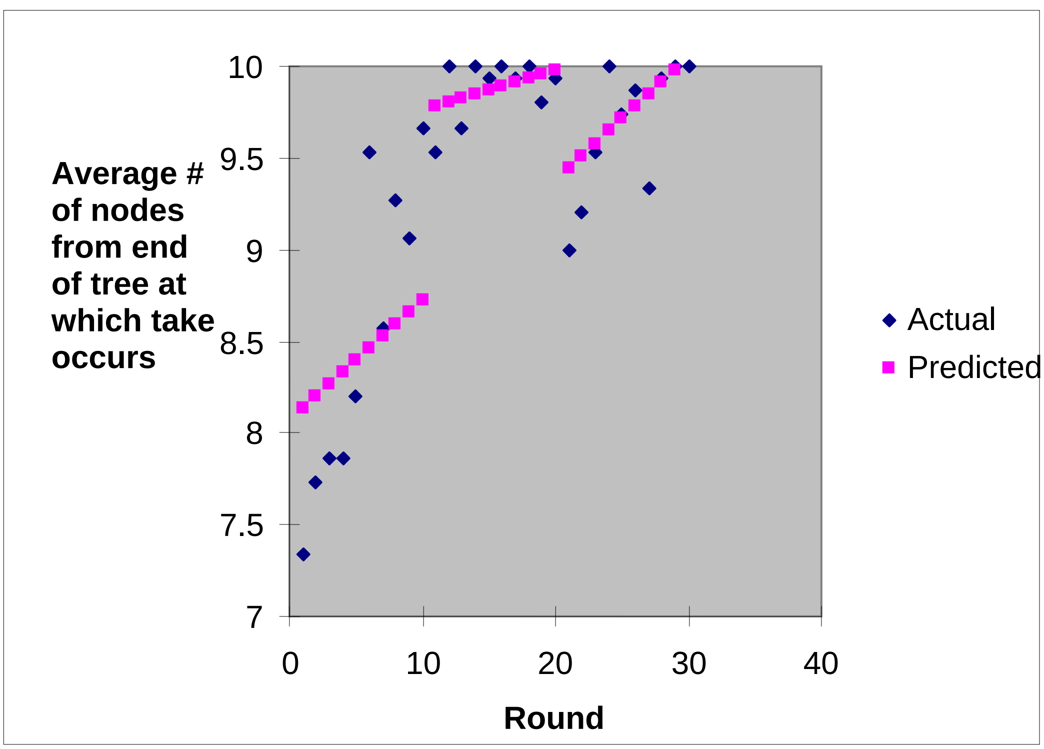
$$\text{Avg. Node of Take} = a + b_1 \times \text{Time} + b_2 \times \text{Format Dummy}$$
$$+ b_3 \times \text{Time} \times \text{Format Dummy} + \varepsilon_t$$

For the T2-T1-T2 case we find that there are significant changes between treatments. An F-test for coefficents jointly zero rejects that the slope and intercept treatment dummies are equal to zero (calculated F = 4.56, critical F = 3.35 at 5% level). Graphically, the line fit tells the story.




Both statistically and visually it appears that switching from the clock format (rounds 1-10) to the tree format (rounds 11-20) is associated with subjects postponing the act of taking. Furthermore, switching back to the clock format from the tree format is associated with a return to earlier takes (in rounds 21-30). While one might claim that the results from rounds 11 through 30 just illustrate unraveling to the subgame perfect equilibrium – and they might – no such explanation can be made for the sharp break between round 10 and round 11, which if anything illustrates "deconvergence" from such an equilibrium.

In a further check on the ability of changes in game form to perturb play on a within-subject basis, the T1-T2-T1 session also reveals significant differences in convergence to equilibrium across treatments. Switching from the tree format (rounds 1-10) to the clock format (rounds 11-20) leads to a break in the series, associated with subjects taking much earlier in the clock format. While this could be argued to be evidence of convergence to the SPE (though such an argument would incorrectly ignore the role of the time trend variable in controlling for such convergence) the switch back to the tree format in rounds 21-30 is associated with a break *away* from the SPE (counter to the time trend); this puts to rest the idea that switching formats does not affect play on a within-subjects basis. Statistically the treatments are significant (calculated F = 12.2151, critical F = 3.35 at 5% level), and graphically the results are as below.



## 7. Summary and Conclusions

We get some striking results. Bids and takes are consistently higher with the clock format (traditionally associated with Dutch auctions) than with the tree format (traditionally associated with centipede games) for two new games: sequential Dutch auctions and centipede games with independent private values. In contrast to the existing centipede game literature, we obtain the equilibrium prediction of unraveling to the first take opportunity almost from the outset when using the clock format. In conformity with existing literature, in treatments using the tree format we obtain more typical results, with subjects failing to play the predicted equilibrium until comparatively many repetitions have taken place. Furthermore, we obtain close to the risk neutral prediction for the Dutch auction – when we use the tree format rather than the traditional clock format. The clock format, on the other hand, yields bids greater than risk neutral theoretical bids. Overall, we get data that more closely resemble theoretical predictions for both Dutch auctions and centipede games when we use the (clock or tree) format that is *not* traditionally associated with that market or game.

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